9.4 Life Sensitivity for Stress Effects

The fatigue crack growth life of structural components is significantly affected by the level of applied (repeating) stress and the initial crack size. This section addresses the effect of applied stress level on any structural component and provides examples whereby relative life estimates can be utilized to facilitate the damage tolerant analysis of structural repairs. Section 9.5 discusses the effect that initial crack size has on the crack growth life of a repaired hole.

The simple method for evaluating the effect of stress level on the fatigue crack growth life is based on the general form of Equation 9.3.14 and an available crack growth life curve for the structural geometry of interest. The general form of Equation 9.3.14 related the life (L_σ) at the current stress level (σ) to the life ($L_{x\sigma}$) at the new stress level ($x\cdot\sigma$) through the equation

$$\frac{L_{x\sigma}}{L_\sigma} = \left(\frac{1}{x}\right)^p = x^{-p} \qquad (9.4.1)$$

As explained in Subsection 9.3.3, Equation 9.4.1 will estimate life in a relative sense for any structural detail if (1) the crack growth life is known for a defined stress history and (2) the flight-by-flight crack growth rate behavior is described by the power law equation

$$\frac{da}{dF} = C\overline{K}^p \qquad (9.4.2)$$

Equation 9.4.1 does not allow one to calculate relative life for changes in crack interval, in crack geometry, or in mission mix (unless a master crack growth curve is available for different mission mixes). The above restrictions do not minimize the extensive usefulness of Equation 9.4.1.

Rewriting Equation 9.4.1 so that it relates the unknown crack growth life ($L_{x\sigma}$) to the known life results in

$$L_{x\sigma} = x^{-p} \cdot L_\sigma \qquad (9.4.3)$$

Equation 9.4.3, in essence, provides a scaling factor that would be applied to the complete crack growth life curve for any structural detail; Figure 9.4.1 illustrates this concept schematically. Note that the life scaling factor (x^{-p}) is independent of the shape of the crack growth life curve.

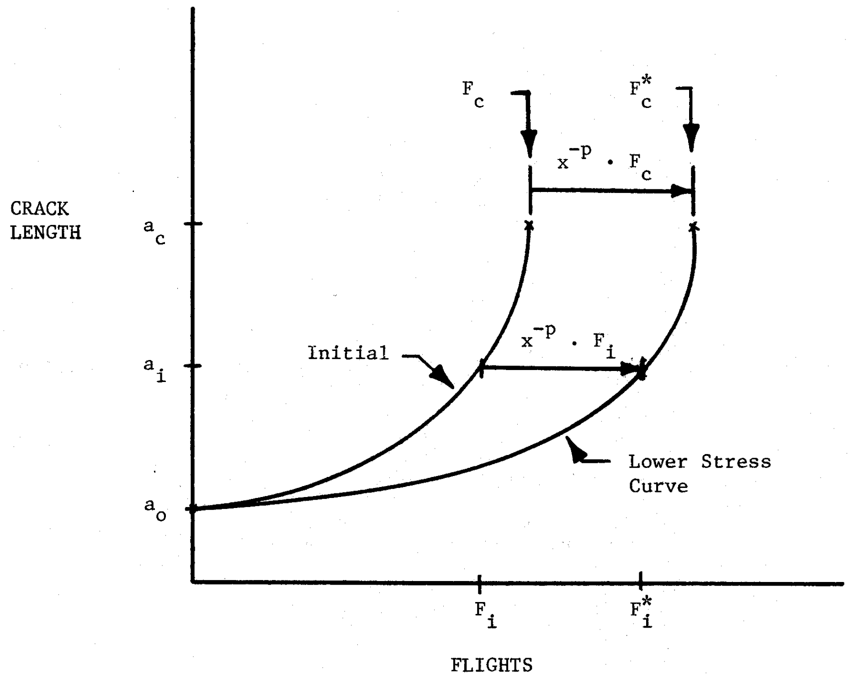


Figure 9.4.1. Schematic Describing the Use of Equation 9.4.3 to Scale the Crack Growth Life Curve Based on a Stress Level Change from σ to $x \cdot \sigma$ where $x < 1$

Due to the generality of the life scaling factor for constructing life estimates, it is instructive to evaluate this factor as a function of the stress scaling factor. The relationship is described in Table 9.4.1 for four different values of the crack growth rate exponent p. Table 9.4.1 shows that the smallest life scaling factors for $x < 1$ are associated with the lowest exponential value ($p = 2.2$). For $x < 1$, the new stress level is lower than the current level and as one would guess (see Table 9.4.1 and Figure 9.4.2), the greater the reduction in stress the longer the life (the higher the life scaling factors).

Table 9.4.1. Relationship Between Stress Scaling Factor x and Life Scaling Factor $L_{x\sigma}$ Defined for Values of the Crack Growth Exponent p

Stress Scaling Factor $x = \dfrac{\sigma_{new}}{\sigma_{current}}$	**Life Scaling Factor (for x^{-p})**			
	p **= 2.2**	p **= 2.5**	p **= 3.0**	p **= 3.0**
0.50	4.60	5.66	8.00	11.31
0.60	3.08	3.59	4.63	5.98
0.70	2.19	2.44	2.92	3.48
0.80	1.63	1.75	1.95	2.18
0.85	1.43	1.50	1.63	1.77
0.90	1.26	1.30	1.37	1.46
0.92	1.20	1.23	1.28	1.34
0.94	1.15	1.17	1.20	1.24
0.96	1.09	1.11	1.13	1.15
0.98	1.04	1.05	1.06	1.07
1.00	1.00	1.00	1.00	1.00
1.02	0.96	0.95	0.94	0.93
1.04	0.92	0.91	0.89	0.87
1.06	0.88	0.86	0.84	0.81
1.08	0.84	0.83	0.79	0.76
1.10	0.81	0.79	0.75	0.72
1.15	0.73	0.71	0.66	0.61
1.20	0.67	0.63	0.58	0.53
1.30	0.56	0.52	0.46	0.40
1.40	0.48	0.43	0.36	0.31
1.50	0.41	0.36	0.30	0.24

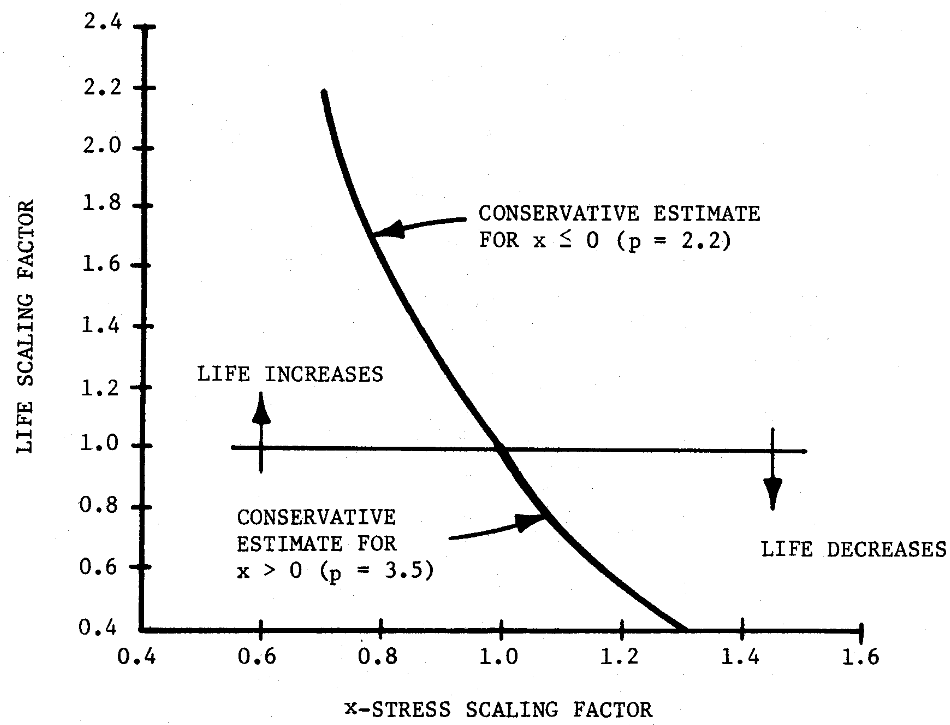
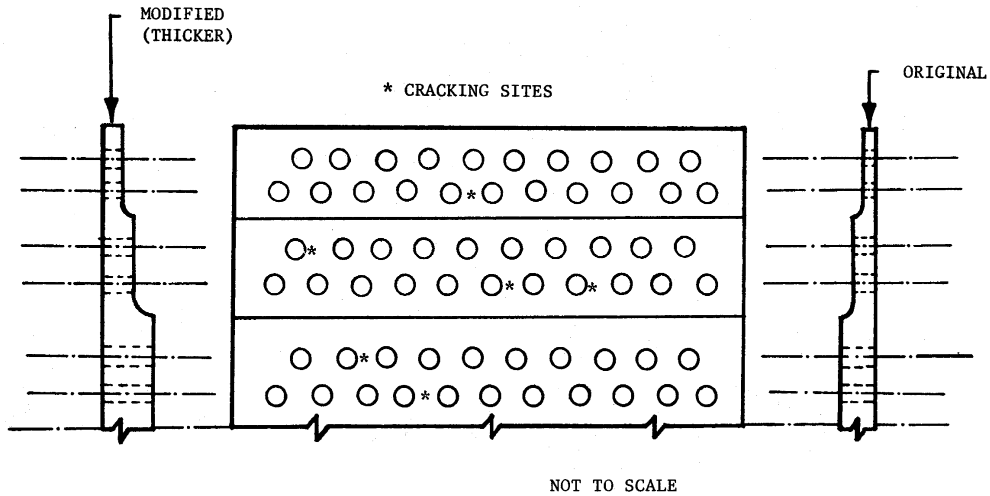


Figure 9.4.2. Life Scaling Factor (New Life / Current Life) as a Function of the Stress Scaling Factor (x = New Stress / Current Stress)

The life benefit achieved by reducing the general level of stress in a structural detail that has experienced crack problems can be estimated from Equation 9.4.3. If the power law exponent p is not available for this particular structural detail, it is recommended that a conservative estimate of p be made, i.e. for a stress reduction chose $p = 2.2$, and evaluate the increase in life on this basis.

EXAMPLE 9.4.1 Modify to Achieve Lower Stress Levels

The doubler shown below has been modified to reduce the general level of stress at the cracking site identified by ten (10) percent. The original doubler on a 6000 hour aircraft had a mean service life of 3400 flight hours to a crack size which would functionally impare the use of this aircraft. How much life will the replacement doubler have? No crack growth life curve exists for the doubler nor for the general area of the wing where it is located. A wide area master curve for the wing is described by a power law equation with exponent $p = 2.89$.



SOLUTION:

The aircraft is presumed to fly the same type of missions with the same frequency after the repair modification as before. Since a master crack growth rate curve is available for the wing, the analyst would evaluate the life of the repair using Equation 9.4.3 with a power law exponent of 2.89. The modification is expected to result in a new life (L_{new}) for a ten (10) percent reduction in stress level (the new stress level is 0.9 times the current stress level). The new life is given by

$$L_{new} = x^{-p} \cdot L_{current}$$

when $L_{current}$ is equal to 3400 flight hours, this reduces to

$$L_{new} = (0.9)^{-2.89} \cdot (3400)$$

$$= 1.356 \cdot 3400$$

$$= 4610 \text{ flight hours}$$

Thus, a first order estimate indicates the life of the replacement doubler will be 35 percent greater than the original doubler. If the original doubler are removed at 2500 hours and replaced with the doubler with the lower stress, it is anticipated that the replacement doubler will not fail during the remaining life of the aircraft (2500 + 4610 7110 hours > 6000 hour life requirement).

If no information on the crack growth rate behavior existed for this region where the doubler was located, then it is suggested that the equation be evaluated with $p = 2.2$. The result of this evaluation is 4285 hours, which still indicates that the replacement doubler will outlast the aircraft (2500 + 4285 = 6785 hours > 6000 hour life requirement).

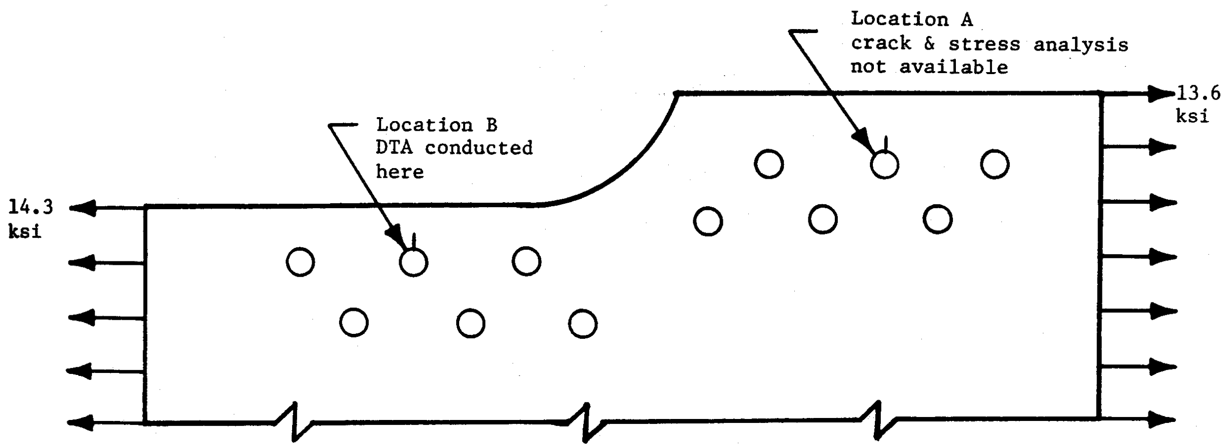
As a cautionary note, it is important to recognize that the best estimate of the exponent p will result in the best life estimate. The exponent p is expected to vary as a function (due to material and stress event effects on damage) so if values of the exponent p are available for a given location in a component, it is more accurate to utilize the exponent p for that location.

Another direct application of Equation 9.4.4 comes from moving from a stress analysis control point where a complete crack growth life analysis is available to a new location where the cracking behavior is expected to be similar due to geometrical material conditions, but where only a strength of materials analysis is available. An example illustrates the approach here.

EXAMPLE 9.4.2 Local Stress Scaling

The figure describes a local area (Location A) of an aircraft structure that has been experiencing distress. Only the most critical hole (Location B) in the region was analyzed during a damage tolerance analysis; this analysis is summarized in the figure. The exponent p associated with the aircraft's standard operational missions is 3.2 for location B.

A strength of materials analysis was conducted to evaluate the difference in stress levels at the two location (A & B) for a given external loading; these stress levels are defined in the geometry. Provide an estimate of the life for the hole identified at Location A.



Description of Structural Geometry and Definition of Analysis Location and of Crack Site

SOLUTION:

The crack at Location A is presumed to grow in the same manner illustrated for Location B. The stress history at Location A is identical to that at Location B except that the stresses are scaled to a lower level x given by

$$x = \frac{\sigma_A}{\sigma_B} = \frac{13.6}{14.3} = 0.951$$

So that the life (L_A) at Location A is found using Equation 9.4.3 and the crack growth life curve in Figure 9.4.5, which describes the life (L_B) to any given crack size for location B:

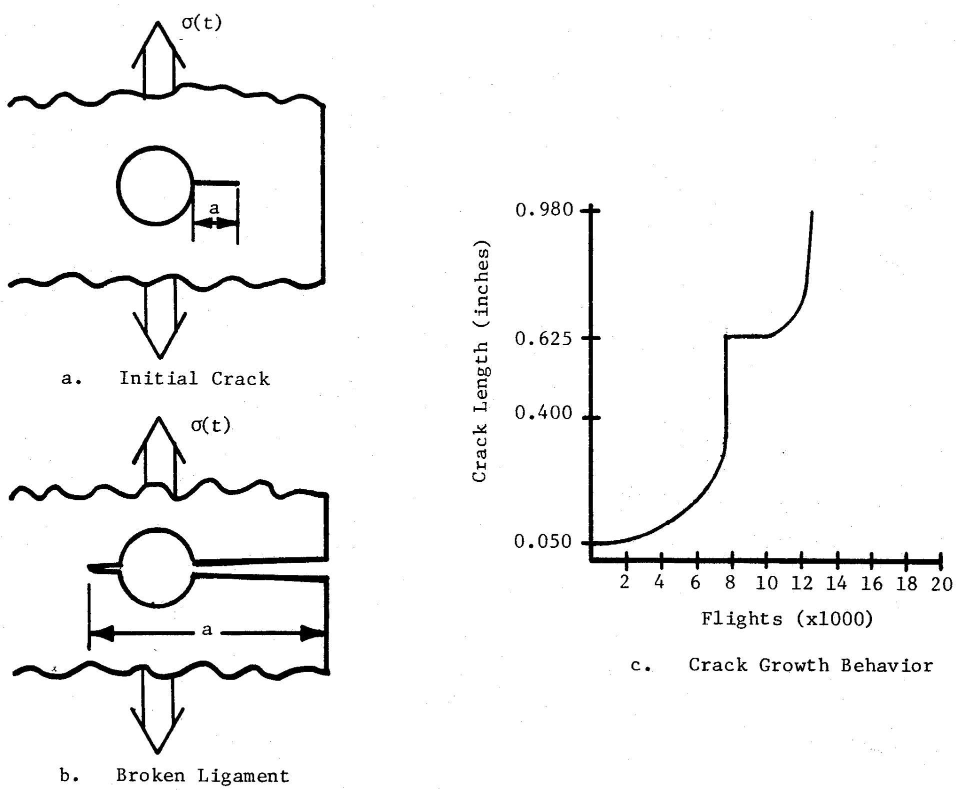
$$L_A = (0.951)^{-3.2}(L_B)$$

From the Location B crack life curve, the flights required to break the ligament and to fracture the component are 7300 and 12100 flights, respectively. From the equation, the corresponding lives at Location A are 8570 and 14210 flights, respectively, a 17 percent over that of location B.

If cracks are observed with a greater frequency at location A than at Location B, and if the crack sizes at location A are longer than that anticipated at location B for the same operational conditions, then the analyst might reverse the analysis, i.e. use the life ratios for specific crack sizes to obtain a better indication of the stresses at the distressed location.

EXAMPLE 9.4.3 Stress Estimated from Crack Behavior

Cracks have been noted during PDM in a number of aircraft at Location A show for Example 9.4.2. From the available inspection data, it appears that the cracks reach a length of 0.150 inches after about 3600 flights. The DTA established crack growth life curve indicated that 0.150 inch long cracks should not appear until 5800 flights. Estimate the stress level difference between location A and B. Also estimate the number of flights required to fail the ligament and the component.



Details of Cracking Process at Location B and Life Curve

SOLUTION:

The method suggested for determining the stress level difference is with Equation 9.4.3, i.e.

$$L_A = x^{-p} \cdot L_B$$

where it's known that $L_A = 3600$, $L_B = 5800$, and $p = 3.2$. Solving for x, the stress ratio between Location A and B yields

$$x = \frac{(\sigma_A)}{(\sigma_B)} = \frac{(L_B)^{\frac{1}{p}}}{L_A}$$

and the stress ratio is

$$x = \frac{(5800)^{\frac{1}{3.2}}}{3600} = 1.16$$

So the stresses at the cracking site (Location A) are expected to be 16 percent greater than that at the DTA location (Location B).

Equation 9.4.6 can be now used to estimate the lives to grow the crack (at Location A) to fail the ligament and the component with x known, the lives are given by

$$L_A = (1.16)^{-3.2} L_B$$

And with L_B = 7300 and 12100 flights for the Location B critical conditions, L_A = 4540 and 7525 flights, respectively, to fail the ligament and the component at Location A.