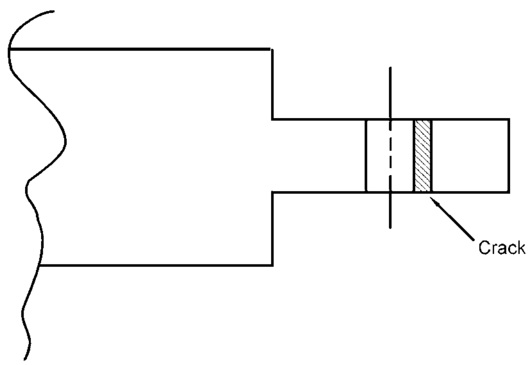
6.3 Slow Crack Growth Structure

The purpose of this example is to demonstrate the lowest level of damage tolerance analysis that can be undertaken. This example problem will be set up to use only a hand-held calculator for all calculations. Some simplifying assumptions to obtain engineering estimates will also be demonstrated.

EXAMPLE 6.3.1 Wing Attachment Fitting

Problem Definition

A training aircraft has been discovered to have cracks in the wing attachment fitting. A redesign and retrofit will be necessary. Cracks have been found in two aircraft that have been grounded. The problem is to determine inspection intervals for the remainder of the force until the modifications can be performed.



Wing Attachment Fitting

Material Property Data

The material for the attachment fitting is 7079 aluminum forging with the following properties:

$K_{Ic} = 22.5 \text{ ksi}\sqrt{\text{in}}$

and a Forman equation describes the crack growth rate behavior:

$$\frac{da}{dN} = \frac{2.25\times10^{-7}(\Delta K)^{3.0}}{(1-R)*22.5-\Delta K}$$

Structural Loads and Stress History

Each aircraft is equipped with a counting accelerometer. The data has been collected and published in the form of n_z counts per 500 hours, as shown in the table.

The stress analysis for the aircraft gives the l-g stress as 7.0 ksi., and using this, the n_z values are converted to stresses. Assuming the 1-g stress is the minimum stress, the stress ratios R can be calculated. These values are also shown in the table.

Stress History for 500 Hours

n_z	Counts/500 Hours	S_{max} (ksi)	R
5.1	80	35.7	0.20
4.5	1200	31.5	0.22
3.5	2500	24.5	0.29
3	12500	21.0	0.33
2	22000	14.0	0.50

Initial Flaw Sizes

The structure is assumed to be slow crack growth structure. A special inspection program has demonstrated an initial flaw size inspection capability of 0.02 inches.

Geometry Model

The critical configuration is determined to be a radial through flaw at the edge of the hole. The stress-intensity factor for this geometry, while well known, is not amenable to closed form solutions. However, applying the approximation techniques discussed in Section 11 leads to an approximate expression for *K* as follows:

$$K = 3\sigma_{max}\sqrt{\pi a}$$

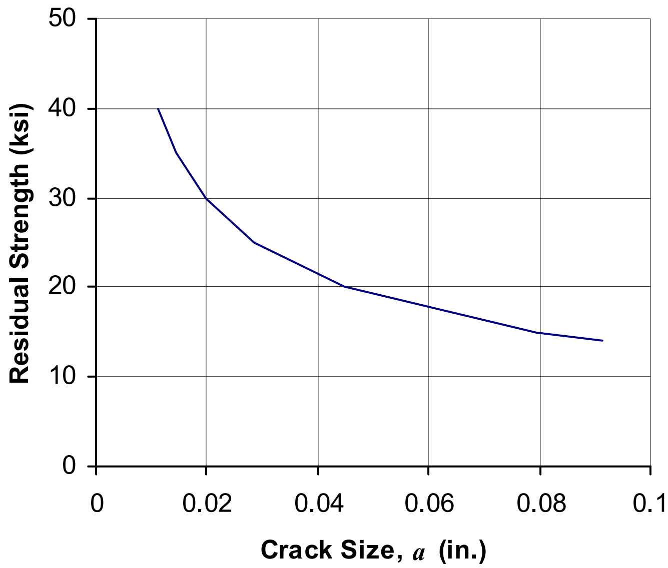
This equation represents a *K* solution for a through crack in a plate multiplied by the stress concentration factor, K_t, for a hole. Using this expression the initial *K*'s for each load level are determined, as shown in the table.

Residual Strength Diagram

The residual strength diagram for this configuration is obtained simply by setting *K* in the above equation equal to K_{Ic} and solving for *a*, which gives:

$$a_{cr} = \frac{K_{I_c}^2}{9\pi\sigma^2}$$

Plotting this function gives the residual strength diagram, as shown.



Residual Strength Diagram

Fatigue Crack Growth Analysis

The basic purpose of this analysis is simply to determine the life under the given stress history. Since the shape of the crack growth curve is not of prime importance because of the imminent retrofit, a damage index approach can be used to estimate the life. The Forman Equation may be integrated to give the life from an initial crack size to critical crack size for n_z level.

$$\int_0^{N_f} dN = \int_{a_0}^{a_f} \frac{(1-R)22.5 - \Delta K}{2.25\times10^{-7}(\Delta K)^{3.0}}\, da$$

Performing this integration gives:

$$N_{allow} = \frac{2}{2.25\times10^{-7}\pi(1-R)^2\sigma_{max}^2}\left[\frac{K_c}{K_o} - 1 - ln\frac{K_c}{K_o}\right]$$

This function is evaluated to give N_{allow} for each stress level in the history. The results are shown in the next table.

Using a fatigue damage analogy, a damage index (DI) is calculated for each stress level by dividing the number of counts in 500 hours by N_{allow}. For n_z = 5.1, the damage index is:

$$DI = \frac{80}{2320.9} = 0.034$$

The life is then obtained by dividing 500 hours by the sum of the damage indices:

$$Life = \frac{500}{\sum DI} = \frac{500}{1.03} = 486$$

Inspection Intervals

The life calculated from the previous analysis is 486 hours under the given usage. Based on the simplifying assumptions made in the analysis an inspection interval of 200 hours until the retrofits are made should be recommended.

Crack Propagation Analysis Using Linear Damage Indices

n_z	Count/500 Hours	S_{max} (ksi)	R	K_o (ksi√in)	N_{allow}	Damage Index
5.1	80	35.7	0.20	8.49	2320.92	0.034
4.5	1200	31.5	0.22	7.49	4260.63	0.282
3.5	2500	24.5	0.29	5.83	13957.88	0.179
3	12500	21.0	0.33	4.99	28875.60	0.433
2	22000	14.0	0.50	3.33	222173.42	0.100
						Sum = 1.027