

4.5 Built-Up Structures

Built-up structures normally require more than one failure criterion to determine the residual strength of the total structure. The development of the residual strength diagram of a given structure will involve the analysis of failures of each part of the load support system.

The structural configuration essentially determines the complexity of the residual strength analysis. Typical structural parameters which must be considered for skin-stiffened structure are:

- Type of Construction
 - Monolithic (unreinforced/forgings)
 - Skin (longerons, stringer)
 - Integrally stiffened
 - Planked
 - Layered (honeycomb/laminated)
- Panel Geometry
 - Planform
 - Curvature
 - Stiffener spacing and orientation
 - Attachments (spar caps, webs, frames, etc.)
- Details of Construction
 - Stiffener geometry (hat, Z-channel, etc.)
 - Attachment details (bolted, riveted, welded, etc.)
 - Fastener flexibility
 - Eccentricity

Ideally, the residual strength analysis will take all these parameters into consideration. In practice, many are treated empirically and others are not considered except in extremely detailed analyses. This section provides details of the analysis methods used for built-up skin stringer structure and the effects of many of the structural parameters listed above. In the order of their presentation, the subsections provide: overviews of the analysis for edge stiffened and for centrally stiffened skin structure, the analysis methods used to determine the stress-intensity factor in the skin structure and the loading transferred to the stringers, the analysis of stiffener failure, the analysis of fastener failure, the analysis methodology and an example.

4.5.1 Edge Stiffened Panel with a Central Crack

The residual strength diagram of a simple panel with two stringers and a central crack can be constructed as follows. Consider first a crack in plane stress, which starts propagating slowly at $\sigma_o = K_{onset} / \sqrt{\pi a_o}$ and becomes unstable at $\sigma_c = K_c / \sqrt{\pi a_c}$ in a sheet without stringers as shown in [Figure 4.5.1a](#).

When the panel is stiffened with stringers, the stress-intensity factor is reduced to $K = \beta \sigma \sqrt{\pi a}$ where $\beta < 1$. As a result, both the stress for slow stable crack growth, σ_o , and the stress for

unstable crack growth, σ_f , are altered to give $\sigma_o = K_{onset} / \beta \sqrt{\pi a_o}$ and $\sigma_{cf} = K_c / \beta \sqrt{\pi a_c}$, respectively.

Hence, these events take place at higher stresses in the stiffened panel than in the unstiffened panel. This means that the lines in [Figure 4.5.1a](#) are raised by a factor $1/\beta$ for the case of the stiffened panel, as depicted in [Figure 4.5.1b](#). Since β decreases as the crack approaches the stringer, the curves in [Figure 4.5.1b](#) turn upward for crack sizes on the order of the stringer spacing.

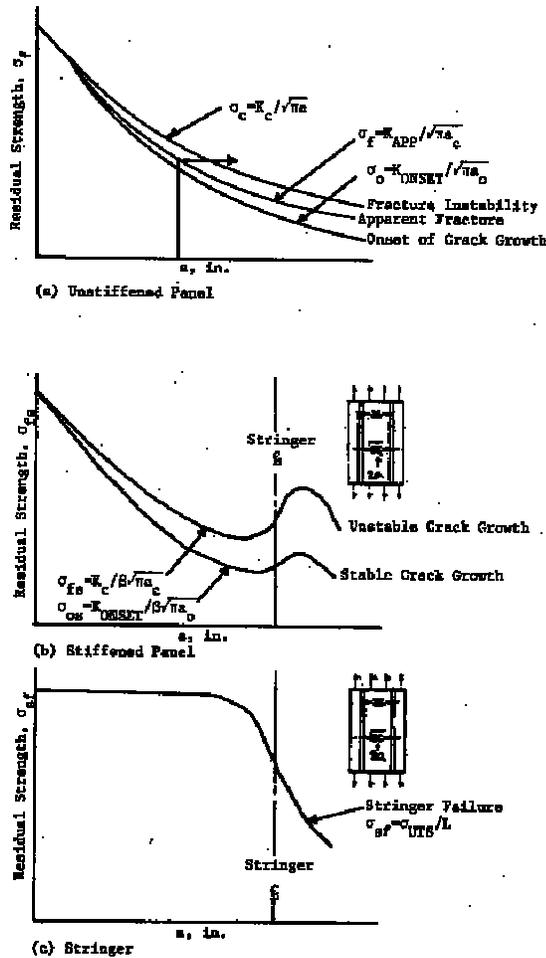


Figure 4.5.1. Elements of Residual Strength Diagram

The possibility of stringer failure should be considered also. The stringer will fail when its stress reaches the ultimate tensile stress (σ_{UTS}). As the stringer stress is $L\sigma$, where σ is the nominal stress in the panel away from the crack, failure will occur at σ_{sf} , given by $L\sigma_{sf} = \sigma_{UTS}$. Using L , a measure of the load transferred to the stringer, the panel stress at which stringer failure occurs is shown in [Figure 4.5.1c](#). The stringer may yield before it fails. This means that its capability to take overload from the cracked skin decreases. As a result, β will be higher and L will be lower. The stress-intensity analysis should account for this effect.

[Figure 4.5.2](#) shows the residual strength diagram of the stiffened panel. It is a composite of the critical conditions shown in [Figure 4.5.1](#). In the case when the crack is still small at the onset of

instability ($2a \ll 2s$, where $2s$ is stringer spacing), the stress condition at the crack tip will hardly be influenced by the stringers and the stress at unstable crack growth initiation will be the same as that of an unstiffened sheet of the same size (Point B in [Figure 4.5.2](#)). When the unstably growing crack approaches the stiffener, the load concentration in the stiffener will be so high that the stiffener fails (Point C) without stopping the unstable crack growth (line BC).

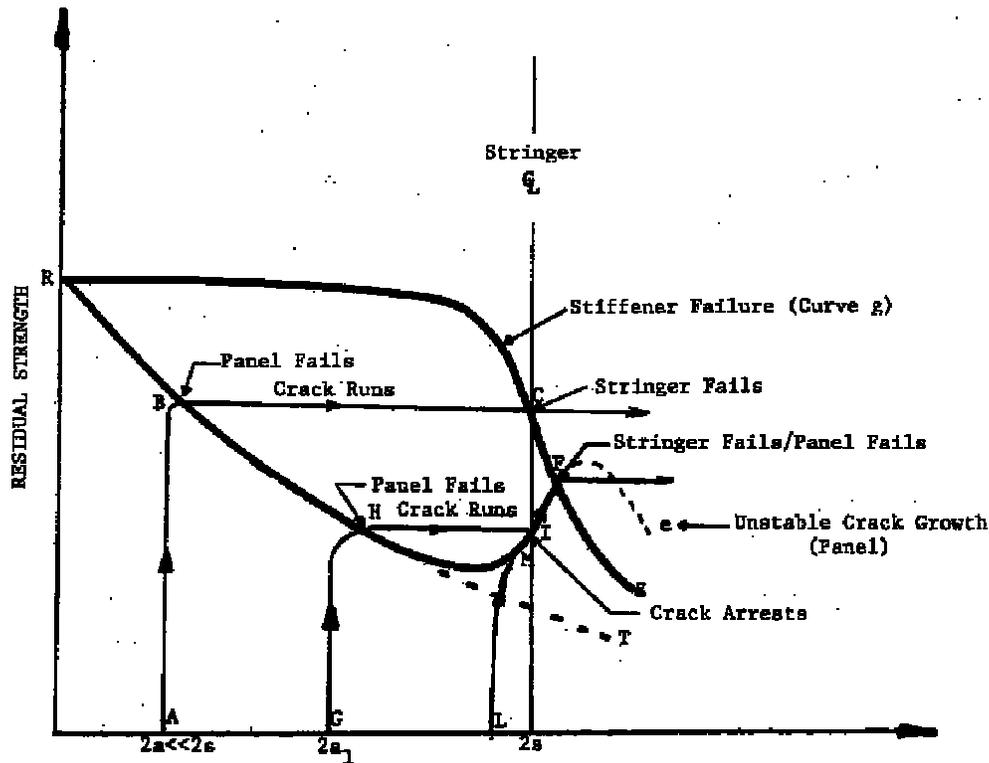


Figure 4.5.2. Residual Strength Diagram for a Stiffened Panel

When the panel contains a crack extending almost from one stiffener to the other ($2a \cong 2s$), the stringer will be extremely effective in reducing the peak stress at the crack tips (β small), resulting in a higher value of the stress at crack growth initiation. With increasing load, the crack will grow stably to the stiffener (line LMIF) and due to the inherent increase of stiffener effectiveness, the crack growth will remain stable. Fracture of the panel will occur at the same stress level corresponding to the point F due to the fact that the stiffener has reached its failure stress and the stress reduction in the skin is no longer effective after stringer failure.

For cracks of intermediate size ($2a = 2a_1$), there will be unstable crack growth at a stress slightly above the fracture strength of the unstiffened sheet (point H), but this will be stopped under the stiffeners at I. After crack arrest, the panel load can be further increased at the cost of some additional stable crack growth until F, where the ultimate stringer load is reached.

Since β and L depend upon stiffening ratio, the residual strength diagram of [Figure 4.5.2](#) is not unique. [Figure 4.5.2](#) shows the case where stringer failure is the critical event. For other stiffening ratios, skin failure may be the critical event as depicted in [Figure 4.5.3](#). Due to a low stringer load connection, the curve e and g do not intersect. A crack of size $2a_1$ will show stable growth at point B and become unstable at point C. Crack arrest occurs at D from where further

slow growth can occur if the load is raised. Finally, at point E, the crack will again become unstable, resulting in panel fracture. It is, therefore, obvious then that a criterion for crack arrest has to involve the two alternatives of stringer failure and skin failure, and these depend upon the relative stiffness of sheet and stringer.

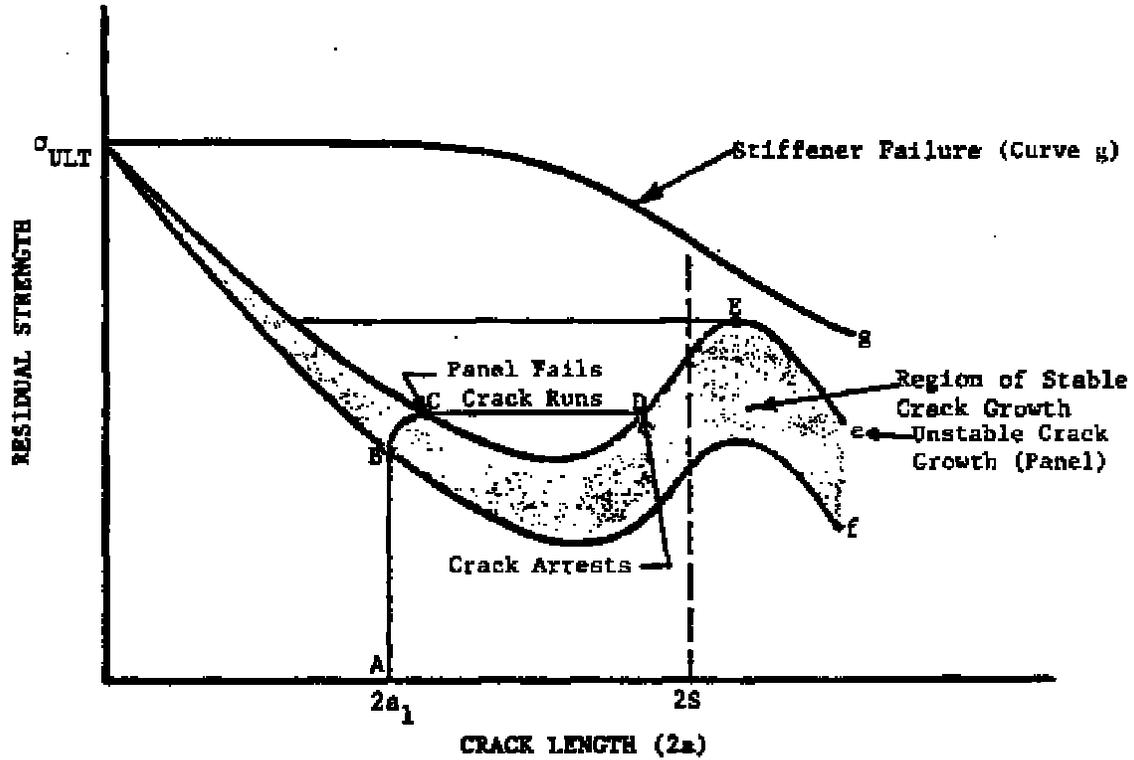


Figure 4.5.3. Panel Configuration with Heavy Stringers; Skin-Critical Case

The foregoing clearly shows that for crack arrest it is not essential that the crack run into a fastener hole. Crack arrest basically results from the reduction of stress-intensity factor due to load transmittal to the stringer.

For the particular case depicted in [Figure 4.5.4](#), the residual strength is not determined by stringer failure solely but also by fastener failure (point K). A crack of length $2a_1$ will show slow growth from E to F and instability from F to G. After crack arrest at G, further slow growth occurs until at point K the fasteners fail. The latter could cause panel failure, but this cannot be directly determined from the diagram.

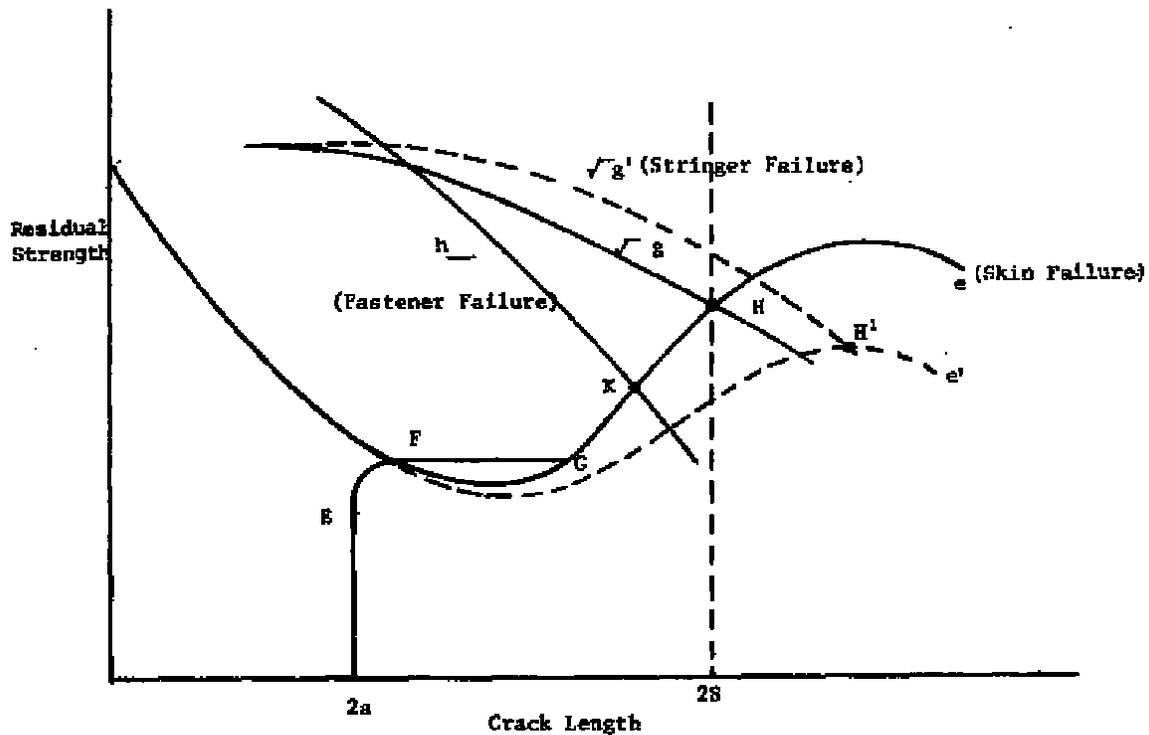


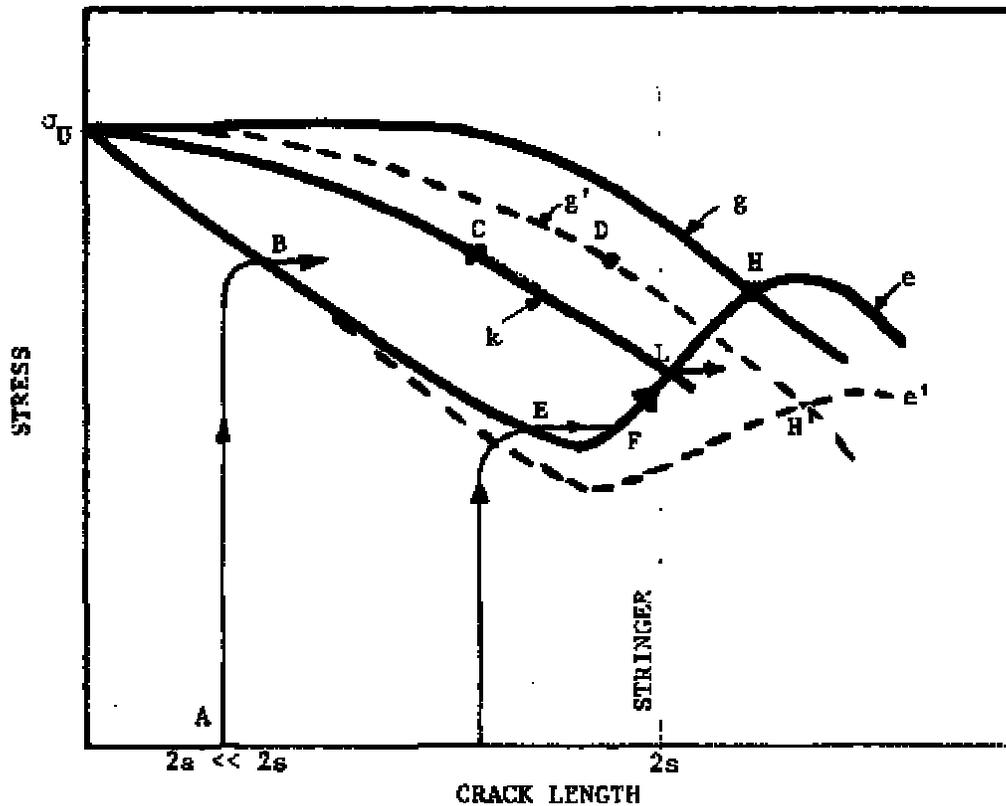
Figure 4.5.4. Criterion for Fastener Failure

In fact, a new residual strength diagram must now be calculated with omission of the first row of rivets at either side of the crack. Fastener failure will affect load transmittal from the skin to the stringer: line e will be lowered, line g will be raised. The intersection point H' of the new lines g' and e' may still be above K and hence, the residual strength will still be determined by stringer failure at H' .

In reality, the behavior will be more complicated due to plastic deformation. Shear deformation of the fasteners, hole deformation, and plastic deformation of the stringers will occur before fracture takes place. Plastic deformation always reduces the ability of the stringer to take load from the skin that implies that line g in actuality will be raised and line e will be lowered. The intersection of the two lines (failure point) will not be affected a great deal, however, (compare points H and H' in [Figure 4.5.4](#)). For this reason the residual strength of a stiffened panel can still be predicted reasonably well, even if plasticity effects are ignored. Nevertheless, a proper treatment of the problem requires that plasticity effects be taken into account.

4.5.2 Centrally and Edge Stiffened Panel with a Central Crack

In the previous subsection, the cases considered pertain to cracks between two stiffeners. In practice, however, cracks frequently start at a fastener hole and then there will be a stringer across the crack which will have a high load concentration factor. The problem can be dealt with in a manner similar to a crack between stringers, using either analytical or finite-element procedures. A schematic residual strength diagram for this case is presented in [Figure 4.5.5](#). Apart from the residual strength curve g for the edge stiffeners, there will now be an additional residual strength curve k for the central stiffener.



- f: Residual Strength Curve of Skin Alone
- g: Residual Strength Curve of Two Edge Stiffeners
- k: Residual Strength Curve of Central Stiffener (Failing).
- g': Residual Strength Curve of Two Edge Stiffeners After the Failure of Central Stiffener

Figure 4.5.5. Residual Strength Diagram for a Panel with Three Stiffeners and a Central Crack Emanating from a Rivet Hole

For the case where the crack in the skin is small ($2a \ll 2s$), the first failure in the structure is noted to occur at point B in [Figure 4.5.5](#) where the skin fails and the crack starts to run. When the crack reaches a size such that point C is reached, the central stiffener residual strength has dropped to the operating stress level and then the central stringer fails, immediately causing

additional loading to be transferred to the edge stiffeners and the skin structure. The effect of losing the capability of the central stringer is noted in [Figure 4.5.5](#) with a repositioning of the residual strength curves from the edge stiffeners (from curve g to curve g') and skin structure (from curve e to curve e'). As the crack in the skin structure continues to grow after causing the ultimate tensile strength failure in the central stringer at point C, it reaches a size that causes the ultimate tensile strength failure of the two edge stringers at point D, at which point all potential arrest capability is lost and the structure is lost.

For the case of longer cracks, [Figure 4.5.5](#) shows that skin cracks may start running (line EF), arrest (point F), and tear along curve FL as the stress is increased. At point L, the crack has reached a length that has resulted in sufficient stress being transferred to the central stringer so that this stiffener now fails. Again, this failure causes a redistribution of stress in the entire structure so that a new set of residual strength diagrams are required to determine the consequences associated with failing the central stringer. The new edge stringer and skin structure residual strength curves are presented by curves g' and e' , respectively.

Due to the high load concentration, the middle stringer will usually fail fairly soon by fatigue and, therefore, lines e' and g' , with the middle stringer failed, will have to be used and the residual strength is determined by point H'. (Note that e' , g' , and H' will have different positions in the absence of the middle stringer; a failed central stringer will induce higher stresses in both the skin and the edge stiffeners.) The foregoing discussion provides the concepts required to establish a complete residual strength diagram.

4.5.3 Analytical Methods

In this subsection analytical procedures are presented for the residual strength capability analyses. Methods for evaluating the unknown fastener force and the stress-intensity factors for the stiffened panel are presented. Since the equations for the solution procedures have been based on linear elastic fracture mechanics, the failure criterion used in these analyses are also based on fracture toughness values for abrupt fracture conditions and K_R resistance curve data for tearing fracture conditions.

Analysis methods for stiffened panels have been developed independently by Romualdi, et al [1957], Poe [1970, 1971], Vlieger [1973], Swift and Wang [1969], Swift [1971], Creager and Liu [1971], and Wilhem and Ratwani [1974].

Application of the stress intensity factor parameter, β , and the stringer load concentration factor, L , were proposed by Vlieger [1973] and Swift and Wang [1969].

From the residual strength capability analysis as discussed in the preceding subsections, it is evident that the construction of residual strength diagrams for built-up structures also requires the estimation of the stress-intensity factor K . A number of approaches for determining K have been developed. Solutions for complicated structural geometries can sometimes be obtained from the basic stress field solutions combined with displacement compatibility requirements for all the structural members involved. This approach has been shown by several investigators to be useful in the analysis of built-up sheet structure. While the analysis is based on closed form solutions, the actual analyses are computerized for efficient solutions. The essentials of this technique are described below.

In calculating β and L , two methods can be used. There are the finite-element method and an analytical method based on closed-form solutions. The analytical method has advantages over the finite-element method in that the effect of different panel parameters on the residual strength of a certain panel configuration can be easily assessed, so that the stiffened panel can be optimized with respect to fail-safe strength. It allows direct determination of the residual-strength diagram. In the case of the finite-element method, a new analysis has to be carried out when the dimensions of certain elements are changed because a new idealization has to be made. An advantage of the finite-element analysis, on the other hand, is that such effects as stringer eccentricity, hole deformation, and stringer yielding can be incorporated with relative ease. Details of the calculations can be found in the referenced papers.

The procedure for analytical calculation is outlined in [Figure 4.5.6](#). The stiffened panel is split up into its composite parts, the skin and the stringer. Load transmission from the skin to the stringer takes place through the fasteners. As a result, the skin will exert forces F_1, F_2 , etc., on the stringer, and the stringer will exert reaction forces F_1, F_2 , etc. on the skin. This is depicted in the upper line of [Figure 4.5.6](#).

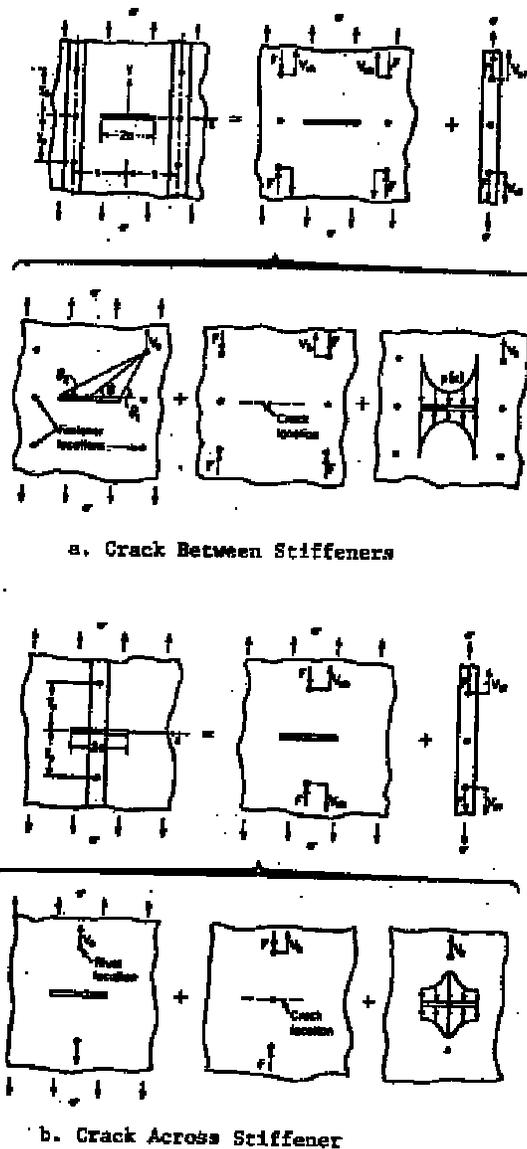


Figure 4.5.6. Analysis of Stiffened Panel

The problem is now reduced to that of an unstiffened plate loaded by a uniaxial stress, σ , and fastener forces $F_1 \dots F_n$. This case can be considered as superposition of three others, shown in the second line of [Figure 4.5.6](#). Namely:

- a. A uniformly loaded cracked sheet.
- b. A sheet without a crack, loaded with forces $F_1 \dots F_n$.
- c. A cracked sheet with forces on the crack edges given by the function $p(x)$. The forces $p(x)$ represent the load distribution given by Love [1944]. When the slit CD is cut, these forces have to be exerted on the edges of the slit to provide the necessary crack-free edges.

The three cases have to be analyzed individually. For case a, the stress-intensity factor is $K = \sigma\sqrt{\pi a}$. For case b, $K = 0$. The stress intensity for case c is a complicated expression that

has to be solved numerically. However, once the K value for case c is determined, the stress-intensity factor for the whole stiffened panel can be obtained by adding the K values for cases a and b.

The determination of K requires calculations of fastener forces $F_1, F_2 \dots F_n$. To calculate these forces, the displacement compatibility conditions which require equal displacements in sheet and stringer at the corresponding fastener locations, can be used. These compatibility requirements deliver a set of n (n = number of fasteners) independent algebraic equations from which the fastener forces can be obtained. These equations can be solved numerically using Gauss-Seidal or Gauss-Jordan iterative methods.

The number of fasteners to be included in the calculation depends somewhat upon geometry and crack size. According to Swift [1974] and shown in [Figure 4.5.7](#), 15 fasteners at either side of the crack seems to be sufficient to get a consistent result. Similar results were obtained by Sanga [1974]. Swift's analysis provides a detailed description of how to incorporate nonelastic behavior in this kind of analysis. The method can account for (1) stiffener flexibility and stiffener bending, (2) fastener flexibility, and (3) biaxiality. Stringer yielding, fastener flexibility, and hole flexibility are lumped together in an empirical equation for fastener deflection.

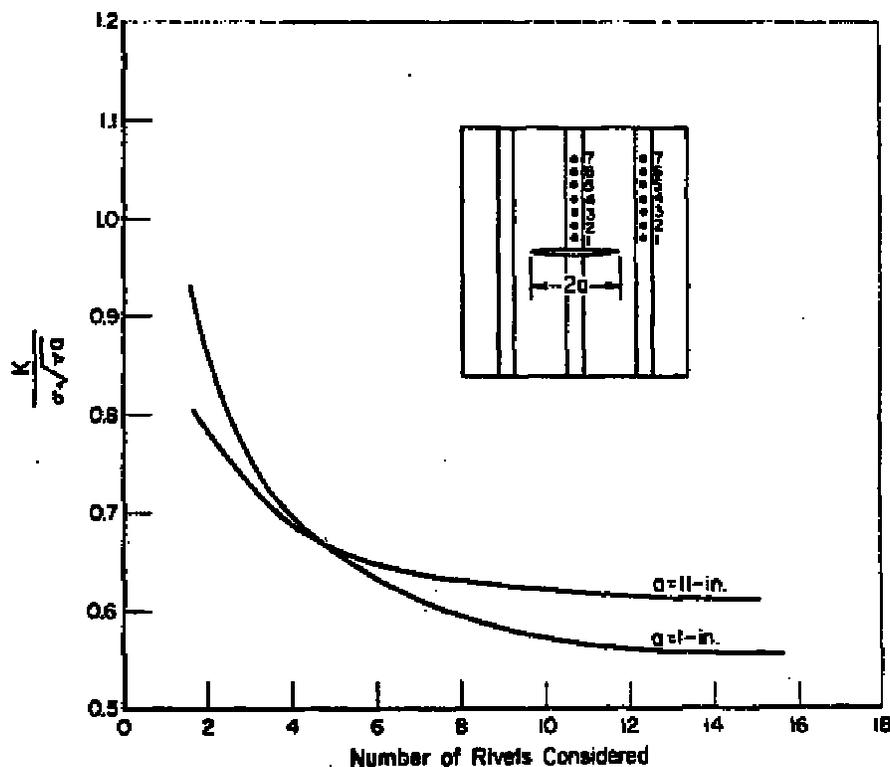


Figure 4.5.7. Effect of Number of Fasteners Included in Analysis on Calculated Stress-Intensity Factor

The effect of fastener flexibility and stiffener bending on β and L is shown in [Figure 4.5.8](#). Although the effects are quite large, the vertical position of the crossover of critical stress-intensity factor curve and stringer stress curve is not affected too much (compare points A and B in [Figure 4.5.8](#)). The level of the crossover determines the residual strength, as pointed out in the

previous subsections. This explains why the residual strength can be reasonably well predicted if the flexibility of the fasteners is neglected.

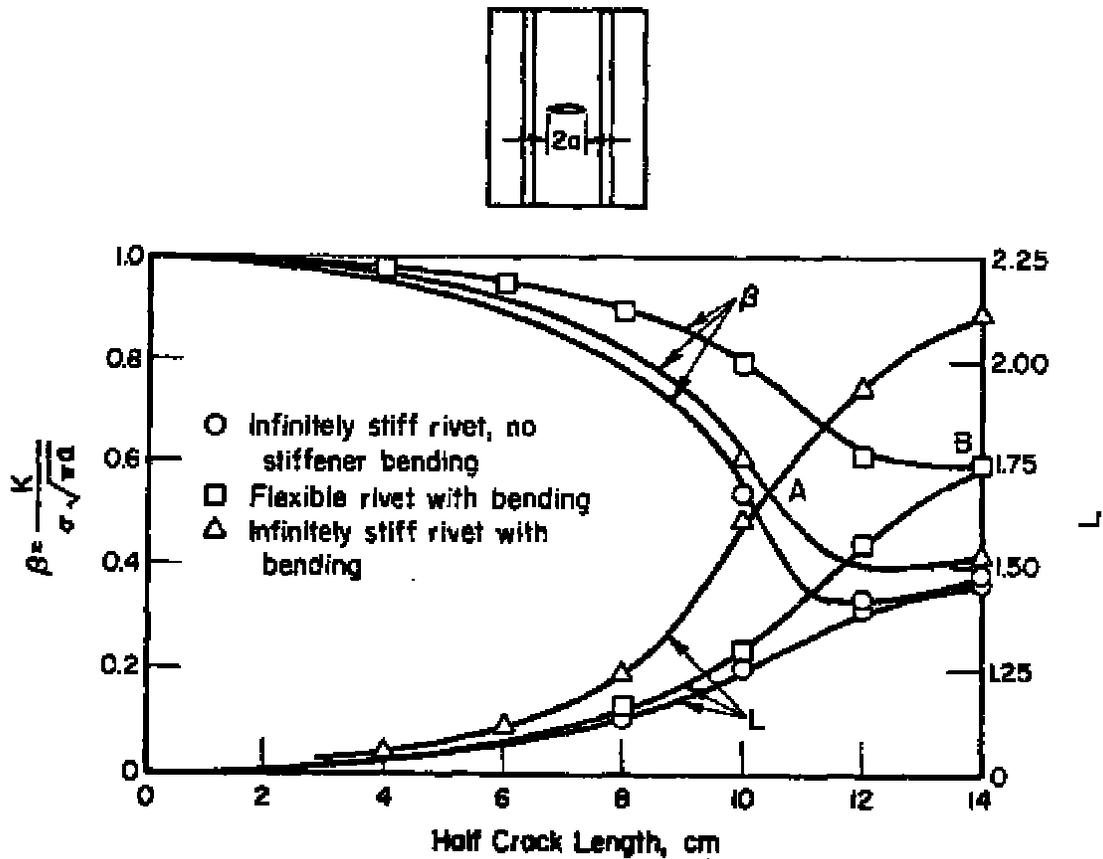


Figure 4.5.8. Skin-Stress-Reduction β and Stringer-Load-Concentration L as Affected by Fastener Flexibility and Stiffener Bending

In the case of adhesively bonded stiffeners, the displacement compatibility approach was used to calculate the fastener loads $F_1, F_2 \dots F_n$. The adhesive was considered by dividing it into a series of discrete segments. The forces $F_1, F_2 \dots F_n$ correspond to the segments shown in [Figure 4.5.9](#). Using an appropriate computational method as explained for riveted fastener, the unknown fastener forces can be evaluated. The method of superposition results in an expression in terms of a complex integral for the stress-intensity factor. A typical residual strength diagram for a bonded structure as compared to the riveted structure is shown in [Figure 4.5.10](#). The required expressions and the solution techniques are discussed in the example problem for a riveted skin-stringer combination with a central crack in the skin.

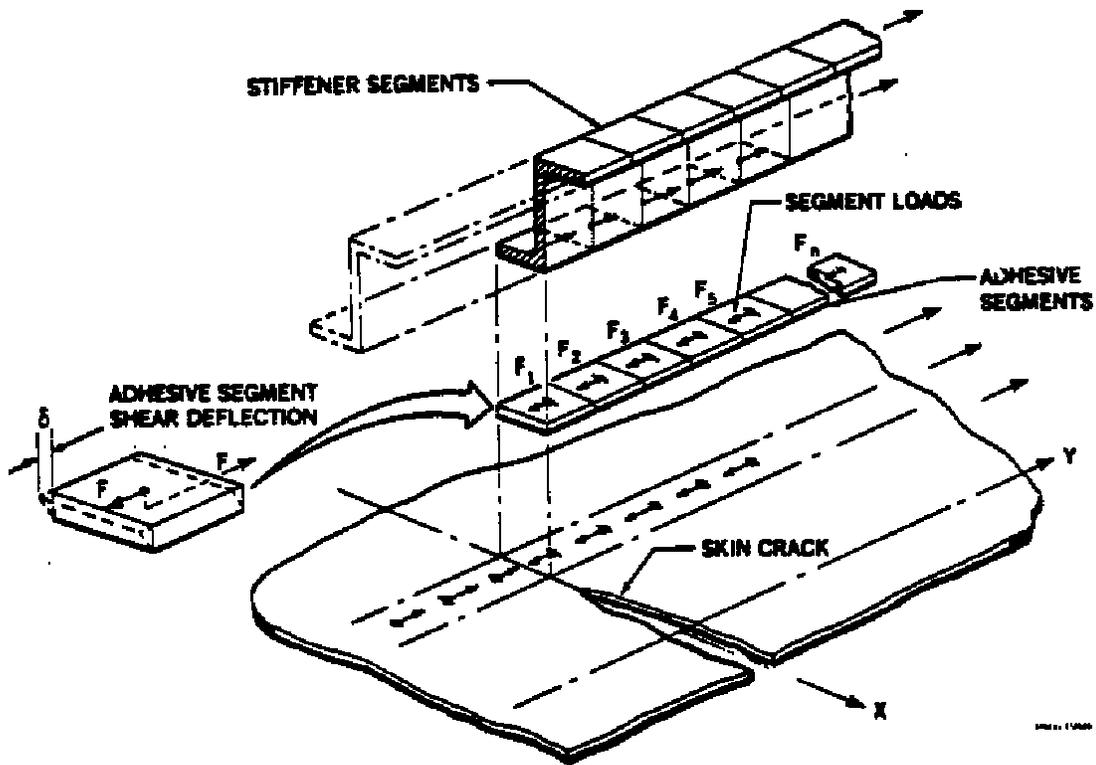


Figure 4.5.9. Bonded Fastener Divided into Discrete Segments

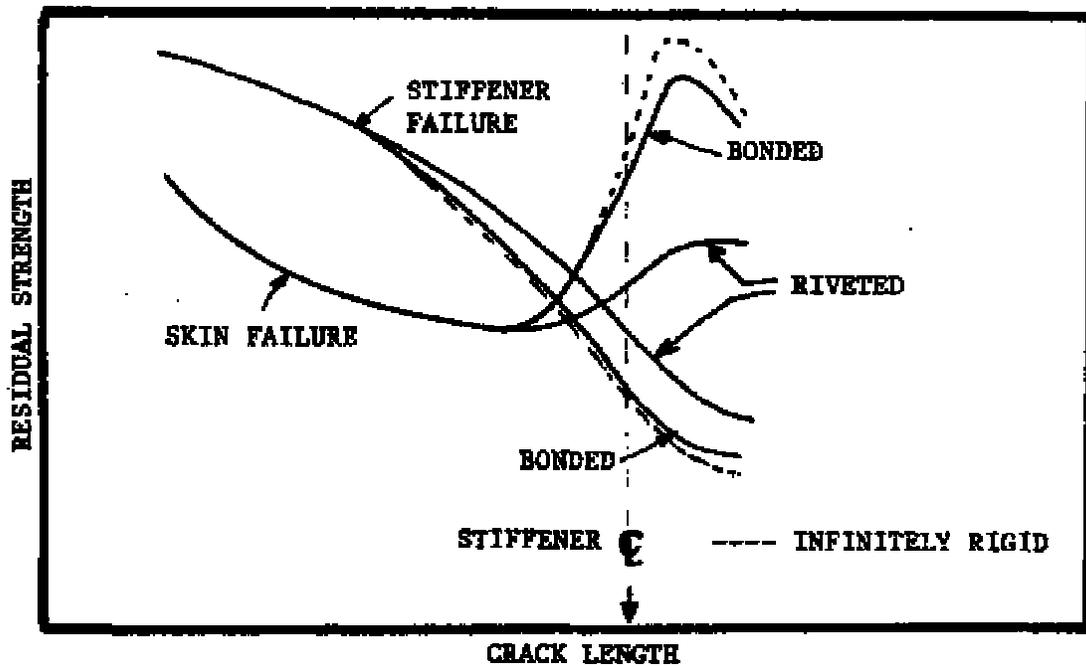


Figure 4.5.10. Residual Strength Diagram Comparing Riveted and Bonded Structures

4.5.4 Stiffener Failure

Stiffener failures are based on the following three stiffener conditions:

1. Intact stiffener (no cracks),
2. Partially failed stiffener (with cracks),
3. Totally failed stiffener.

The failure criterion for the intact stiffener is based on the ultimate strength criterion. As mentioned earlier, the ratio between the stiffener load in the cracked region (P_{max}) and the remote region from the crack (P) is defined as the load concentration factor L_s or

$$L_s = \frac{P_{max}}{P} = \frac{P_{max}}{\sigma A_s} \quad (4.5.1)$$

where σ is the uniform stress in the skin at the loaded end of the panel and A_s is the stiffener cross sectional area. Failure of the stiffener will occur when the value of P_{max} is equal to the ultimate strength of the stiffener (P_{ult}), or when

$$P_{max} = P_{ult} = \psi \sigma_{ult} A_s \quad (4.5.2)$$

where σ_{ult} is the ultimate tensile strength of the stiffener material and $\psi \leq 1$ is a factor accounting for load eccentricity and notch effects in the stiffener. For a uniform stress distribution in the panel remote from the crack the stress in the stringer will equal the nominal stress σ in the skin, i.e.,

$$P = \sigma A_s \quad (4.5.3)$$

Combining equations 4.5.1 to 4.5.2, yields the following stiffener failure criterion:

$$\sigma = \psi \frac{\sigma_{ult}}{L_s} \quad (4.5.4)$$

When the stress in the stringer reaches the value of $\psi \sigma_{ult}$, the stringer will fail. The parameter ψ is determined by tests.

When load eccentricity and notch effects are not considered for a stringer, ψ equals one. The stiffener failure curve obtained using Equation 4.5.4 is shown in [Figure 4.5.11](#). The initial portion of the residual strength curve is flat because the load concentration factor L_s is equal to one for small skin crack lengths. As the skin crack increases in size, L_s becomes significantly greater than one and the stringer carries a large portion of the total structural load which eventually leads to stringer yielding and failure. The portion of the curve in [Figure 4.5.11](#) corresponding to $L_s > 1$ shows the gradual reduction of the residual strength.

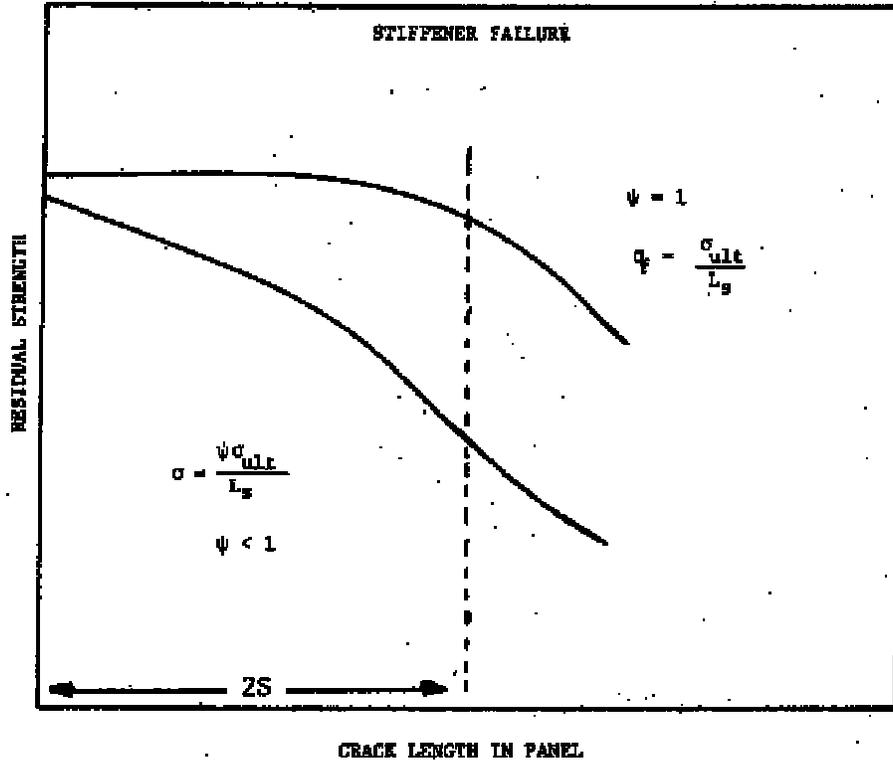


Figure 4.5.11. Residual Strength Diagram for Stiffener

When the load eccentricity and notch effects in the stiffener are considered, the parameter ψ in Equation 4.5.4 is less than one. The residual strength corresponding to a case where $\psi < 1$ is shown in [Figure 4.5.11](#). The curve CD does not have the initial flaw portion exhibited by the case $\psi = 1$. Instead, the residual strength starts decreasing even for small skin crack lengths. The residual strength diagram for the stringer can be constructed knowing the values of L_s and ψ . Determining L_s requires numerical solution techniques that are discussed in the example presented in [subsection 4.5.7](#).

According to JSSG-2006 requirements, cracks are assumed in all load carrying members. This means that all structural elements, stringer included, are assumed to be damaged. The residual strength diagram for the stringer will involve using the fracture mechanics approach of predicting unstable crack growth. The critical stress for a partially cracked stringer is given by

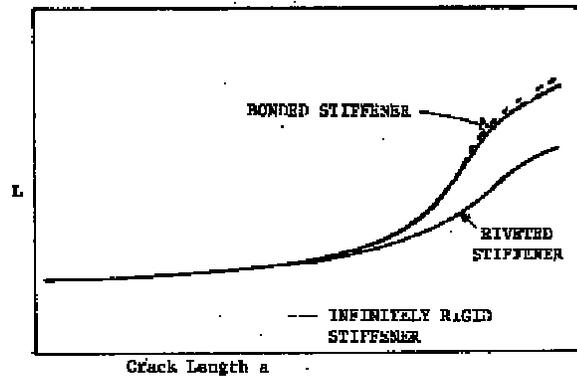
$$\sigma_f = \frac{K_{cr}}{L_s \beta_s \sqrt{\pi a_s}}$$

where K_{cr} is the appropriate fracture toughness, β_s is the stringer geometric

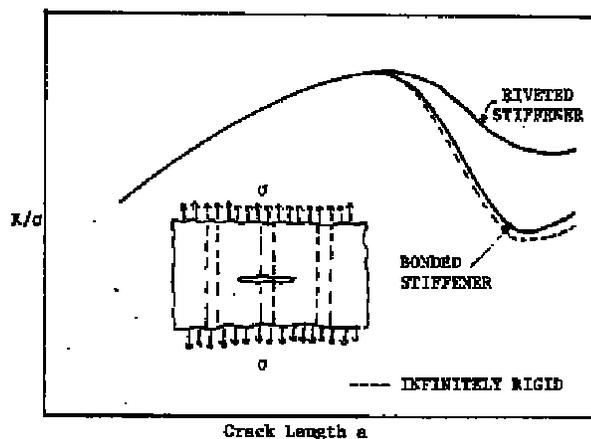
parameter, and a_s is the stringer crack size. When the crack in the panel approaches the stringer, the load transmitted to the stringer will become large ($L_s \gg 1$) and thus the critical stress level required to fail the stringer rapidly decreases as shown by curve CE in [Figure 4.5.11](#). Curve CE corresponds to the total failure of the stringer. This may happen when a large crack emanates from a stringer rivet hole. Total failure of the stiffener occurs before the skin crack approaches the stiffeners.

The residual strength diagram for the stiffened panel in this case would, in fact, be approximately that of the unstiffened panel.

The foregoing discussion presented analysis of a riveted built-up structure. However, built-up structures exist in which the stringer is adhesively bonded to the skin. The load transfer from the skin to the stringer is more effective in the bonded structure due to the increased rigidity in the stiffener. The corresponding load transfer parameter L_s will have higher values as shown schematically in [Figure 4.5.12a](#). Due to the effective load transfer from the skin to the stiffener, the applied stress-intensity factor will be reduced when the panel crack approaches the stiffener. [Figure 4.5.12b](#) illustrates the levels of stress-intensity factor that occur for riveted and bonded stiffeners. The figure also shows that the bonded stiffener is subjected to higher loads due to the effective load transfer; the higher load causes the stiffener failure of the bonded structure to be more critical than that of the riveted structure. [Figure 4.5.13](#) compares the decay of residual strength for these two types of structures. The residual strength of the bonded stiffener decreases faster than the riveted stiffener. In the determination of the residual strength diagram, the parameter L_s is usually calculated by numerical methods. The steps to obtain L_s are discussed later in this section.



(a) Load Transfer Parameter (L)



(b) Stress-Intensity Factor Coefficient (K/σ)

Figure 4.5.12. Comparison of L_s and K/σ for Riveted and Bonded Structures

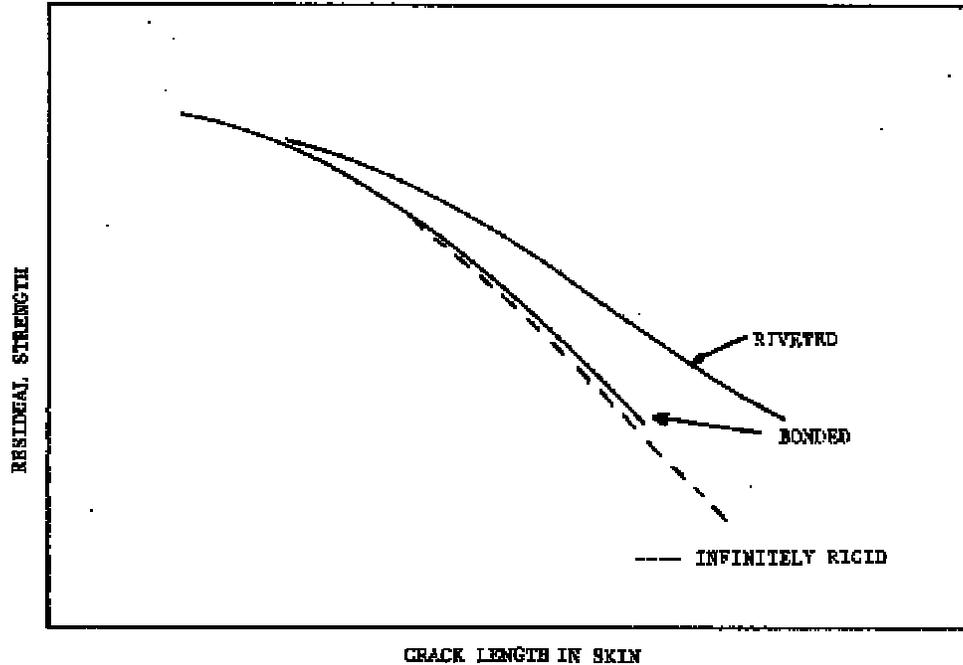


Figure 4.5.13. Comparison of Residual Strength for Riveted and Bonded Stiffeners

4.5.5 Fastener Failure

In [subsections 4.5.3](#) and [4.5.4](#), the discussion focused on skin and stiffener failures. A third mode of failure involves the fasteners. This paragraph will discuss the failure of the fastener system. Load is transmitted from the skin to the stringers through fasteners. If the fastener loads become too high, fastener failure may occur by shear. Fastener failure will reduce the effectivity of the stringer; and therefore, the residual strength of the panel will drop. The highest loads (F) in the stringer/skin connections will occur in the fasteners adjacent to the crack path. Fastener failure will occur when the fastener forces F transmitted by the fasteners adjacent to the crack exceed the critical shear load of the fastener. The fastener failure criterion is given by

$$F = \pi/4 d^2 \tau_{ult} \quad (4.5.5)$$

where d is the fastener diameter and τ_{ult} is the ultimate shear stress of the fastener material. It is emphasized that fastener failure need not necessarily cause total failure of the panel. Once the fastener failure criterion is met, however, the values of L_s and β will change since the loads transferred to the stiffener and skin changes. Once the fastener fails, the values of β and L_s will be recalculated in order to proceed further with the residual strength analysis. The load that causes the fasteners to fail by shear can be calculated from Equation 4.5.5; the corresponding nominal stress in the panel then gives the residual strength curve for the fasteners as shown in [Figure 4.5.14](#). At zero crack length, and for the case where the skin and stringers are made from common materials, the fasteners do not carry any load; the curve therefore tends to increase rapidly for $a \rightarrow 0$. The fastener forces F_i can be computed through the displacement compatibility between the stiffener and the panel. The necessary steps involved in the computation of F_i are discussed in the example presented in [subsection 4.5.7](#).

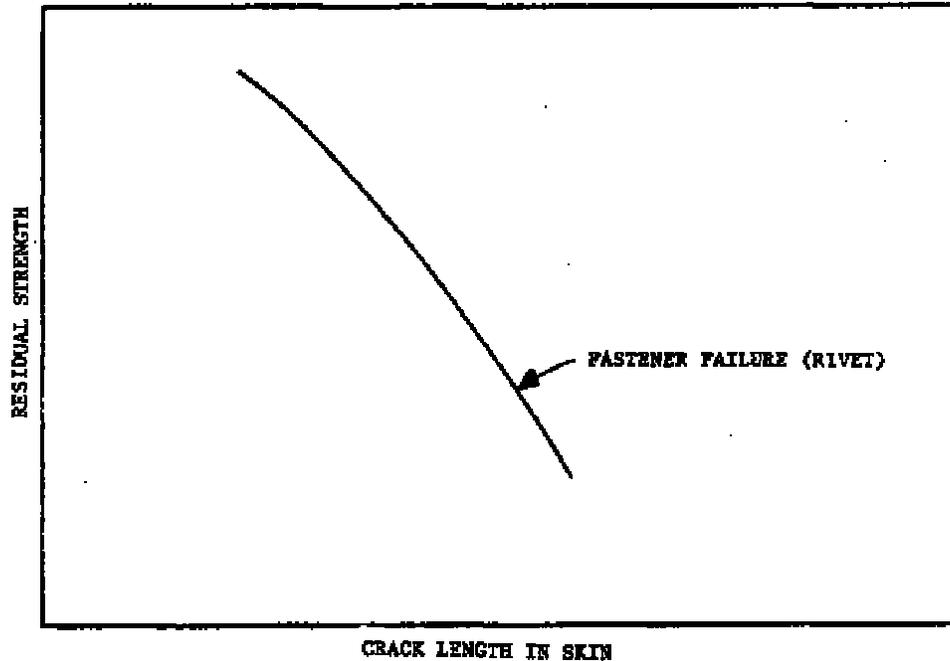
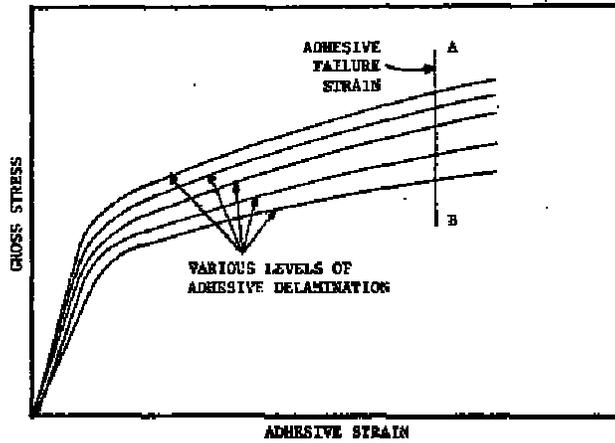
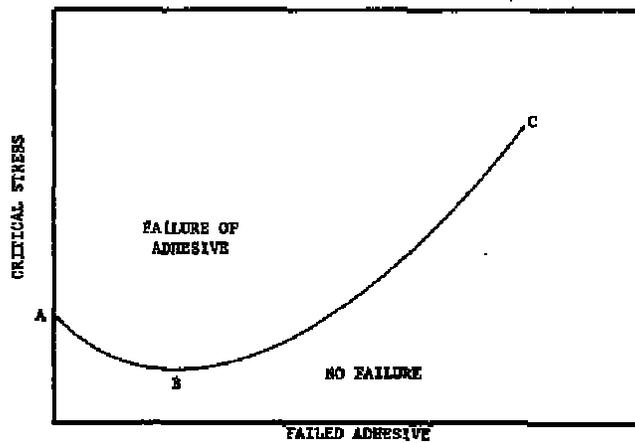


Figure 4.5.14. Residual Strength Diagram for the Fasteners in a Built-Up Structure

In the case of adhesively bonded structures, the adhesive (fastener) failure criterion is based on a maximum adhesive strain value. The residual strength analysis is fairly complicated (see, for example, reference 24). Based on the displacement compatibility between the panel and the stiffener, the adhesive segment strain deflection can be numerically computed for different amounts of disbond. [Figure 4.5.15a](#) shows the adhesive strain versus gross stress for various levels of adhesive delamination. The vertical line AB represents average failure strain of the adhesive. The intersection points between the line AB and the curves give the critical gross stress versus amount of adhesive failed as shown in [Figure 4.5.15b](#). The corresponding curve ABC can be used for panel failure analysis. The area above the curve defines the failure of adhesive.



(a) Stress-Strain Curves for Adhesively Bonded Stringer With Various Levels of Delamination.



(b) Critical Levels of Stress for Adhesively Bonded Stringer.

Figure 4.5.15. Gross Stress and Critical Stress Diagram for Adhesively Bonded Stringer

4.5.6 Methodology Basis for Stiffened Panel Example Problem

The residual strength analysis of an edge stiffened, centrally cracked skin structure of the type shown in [Figure 4.5.16](#) can be performed by following the general steps described in the preceding subsections.

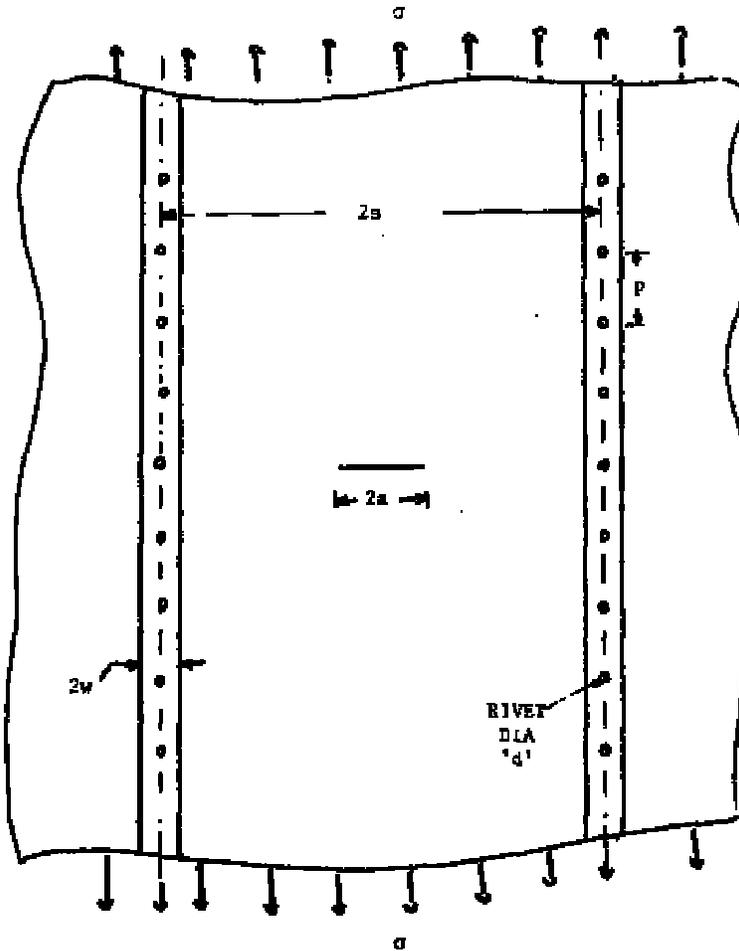


Figure 4.5.16. Riveted Panel with a Central Crack Between Two Stringers

In this subsection, the specific details are covered which are associated with conducting the stress-intensity factor analysis as well as the analysis to determine the stresses in the stringers and fastener loads. To simplify the detailed calculations, it is assumed that only one fastener (rivet) on either side of the crack is active, as shown in [Figure 4.5.17](#) and that this rivet is assumed to be rigid. Thus, there is only one unknown fastener force F transferred between the stringers and the skin by this rivet.

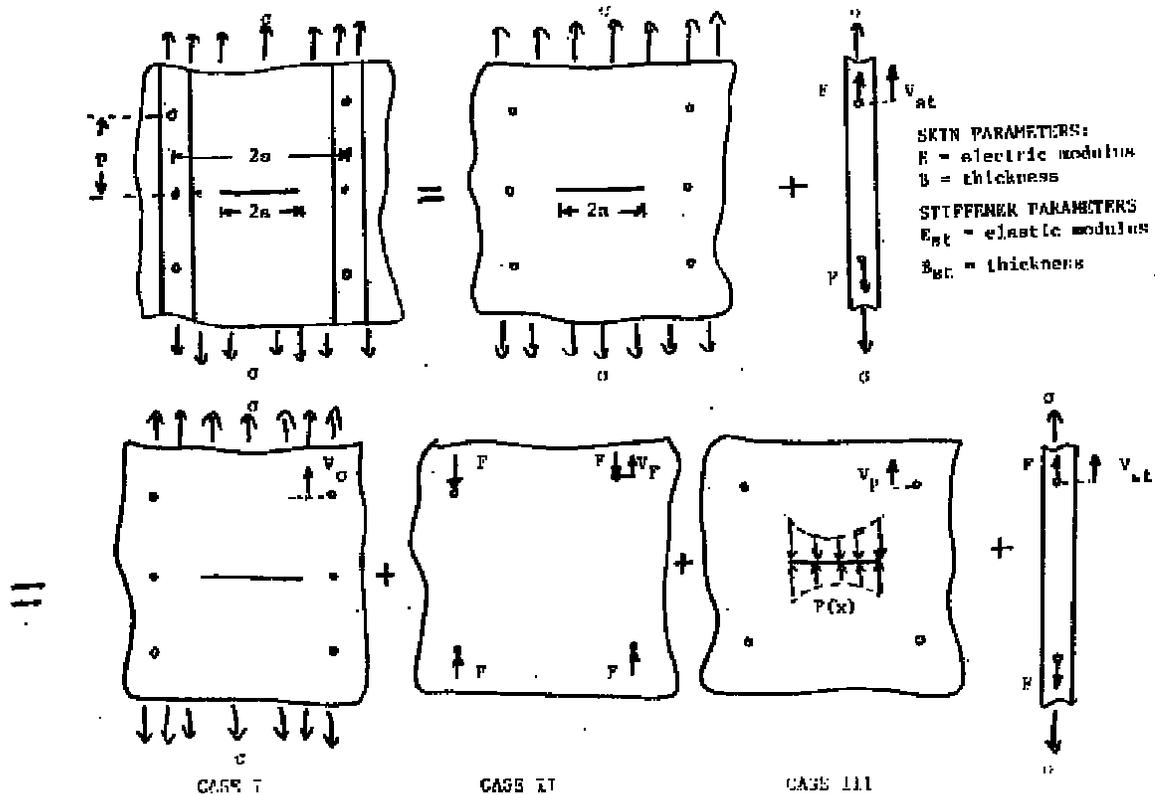


Figure 4.5.17. Stiffened Structure Broken into Components

Typically, the analysis proceeds by splitting up the structure shown in [Figure 4.5.16](#) into its component parts as shown in [Figure 4.5.17](#). The unknown force F can be calculated from the displacement compatibility condition between the skin and the stringer. The complicated expressions which correspond to the displacements V_σ , V_F , and V_P due to the applied stress, σ , the fastener force F and the distributed pressure $P(x)$, respectively, can be obtained using a procedure suggested by Westergaard [1939] and by Love [1944]. The detailed discussions on the methods of obtaining the required relationships are presented by Broek [1974]. The necessary relationships for V_σ , V_F , V_P and V_{st} (displacement in the stringer) are given as:

$$V_\sigma = \frac{\sigma}{E} f_\sigma \tag{4.5.6}$$

$$V_F = \frac{-F}{EB} f_F \tag{4.5.7}$$

$$V_P = \frac{-F}{EB} f_P \tag{4.5.8}$$

$$V_{st} = \frac{F}{E_{st}B_{st}} f_{st} + \frac{\sigma}{E_s} P \tag{4.5.9}$$

where

$$f_{\sigma} = \sqrt{\rho_1 \rho_2} \left\{ 2 \sin \left[\frac{\theta_1 + \theta_2}{2} \right] \frac{(1+\nu)pr}{\rho_1 \rho_2} \cos \left[\theta - \frac{\theta_1 + \theta_2}{2} \right] \right\} + \nu p \quad (4.5.10)$$

$$f_F = \frac{(1+\nu)}{4\pi} \left\{ (3-\nu) \left[\ln \frac{d}{4p} - 1 \right] + (1+\nu) \left[2 - \frac{s^2}{s^2 + p^2} \right] + (3-\nu) \ln \left(\frac{s^2}{s^2 + p^2} \right) \right\} \quad (4.5.11)$$

$$f_P = \frac{\bar{P}}{\pi a} \left\{ 2 \left[\sqrt{\rho_1 \rho_2} \sin \left(\frac{\theta_1 + \theta_2}{2} \right) - p \right] - (1+\nu) p \left[\frac{r}{\sqrt{\rho_1 \rho_2}} \cos \left(\theta - \frac{\theta_1 + \theta_2}{2} \right) - 1 \right] \right\} \quad (4.5.12)$$

where

$$\bar{P} = \left\{ \tan^{-1} \left(\frac{a+s}{p} \right) + \tan^{-1} \left(\frac{a-s}{p} \right) + \frac{p(1+\nu)}{2} \left[\frac{a+s}{p^2 + (a+s)^2} + \frac{a-s}{p^2 + (a-s)^2} \right] \right\} \quad (4.5.13)$$

and

$$f_{st} = -\frac{(1+\nu)}{4\pi} \left\{ \left[(3-\nu) \left(\ln \frac{d}{4y_o} - 1 \right) + (1+\nu) \right] + \right. \quad (4.5.14)$$

$$\left. 2 \sum_{n=1}^{\infty} \left[(3-\nu) \ln \frac{nw}{\sqrt{n^2 w^2 + 4p^2}} + (1+\nu) \left(\frac{4p^2}{n^2 w^2 + 4p^2} \right) \right] \right\}$$

The geometric variables r , ρ_1 , ρ_2 , θ_1 , θ_2 and θ are shown in [Figure 4.5.18](#). The displacement compatibility condition requires equal displacements in corresponding points of sheet and stringer; it yields the following equation to calculate the unknown fastener force F .

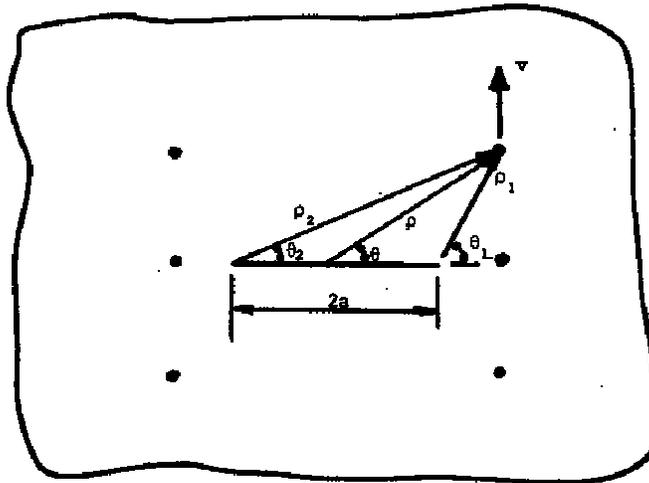


Figure 4.5.18. Geometrical and Displacement Parameters Relative to the Crack Tip

$$V_{\sigma} + V_F + V_p = V_{st} \quad (4.5.15)$$

substituting the expressions 4.5.6 - 4.5.9 for V_{σ} , V_F , V_p , and V_{st} in the above relationship, and reassembling, we get

$$F = \sigma\lambda \text{ where } \lambda = \left\{ \frac{\frac{f_{\sigma}}{E} - \frac{p}{E_{st}}}{\frac{f_F + f_p}{EB} + \frac{f_{st}}{E_{st}B_{st}}} \right\} \quad (4.5.16)$$

The next step is to obtain an expression for the stress-intensity factor for the entire stiffened panel configuration. Using superposition, the stress-intensity factor is obtained as the sum of the stress-intensity factors for the three cases shown in [Figure 4.5.17](#). It can easily be seen that for Case I: $K = \sigma\sqrt{\pi a}$ and for Case II: $K = 0$. The stress-intensity factor (K) for Case III is a fairly complicated expression and it is given by,

$$K_{III} = -2\sqrt{\frac{a}{\pi}} \frac{1}{\pi Bp} \left[\frac{(3+\nu)}{2} I_1 + (1+\nu) I_2 \right] \quad (4.5.17)$$

where

$$I_1 = \int_0^a \frac{d\bar{x}}{\sqrt{a2\bar{x}2}} \left\{ \frac{1}{(1+(\bar{x}-\bar{s})^2)} + \frac{1}{(1+(\bar{x}+\bar{s})^2)} \right\} \quad (4.5.18a)$$

and

$$I_2 = \int_0^a \frac{d\bar{x}}{\sqrt{a2-\bar{x}2}} \left\{ \frac{(\bar{x}-\bar{s})^2}{(1+(\bar{x}-\bar{s})^2)^2} + \frac{(\bar{x}+\bar{s})^2}{(1+(\bar{x}+\bar{s})^2)^2} \right\} \quad (4.5.18b)$$

where \bar{a} , \bar{x} and \bar{s} are normalized with respect to the rivet pitch. The estimation of K_{III} requires solution of the above integrals by numerical methods. Replacing the fastener force F by the expression and rearranging the expression for K_{III} , the stress-intensity factor K for the stiffened panel then becomes

$$K = \sigma\sqrt{\pi a} - \sigma\sqrt{\pi a}\lambda_1\lambda_2 \quad (4.5.19)$$

where

$$\lambda_1 = \frac{(3+\nu)}{2} I_1 + (1+\nu) I_2 \quad (4.5.20a)$$

and

$$\lambda_2 = \frac{2\lambda}{\pi^2 Bp} \quad (4.5.20b)$$

The stress-intensity factor K can be finally expressed in the following form,

$$K = \sigma\beta\sqrt{\pi a} \quad (4.5.21)$$

where

$$\beta = (1 - \lambda_1\lambda_2) \quad (4.5.22)$$

To calculate K for a given stiffened panel the values of F_σ , f_F , f_p , f_{st} , and λ_l have to be obtained. These variables are numerically calculated and plotted as shown in [Figures 4.5.19 to 4.5.23](#) for various values of \bar{s} , \bar{a} , and \bar{d} . For the given example data, we can now construct the residual strength diagram using the values obtained from these plots.

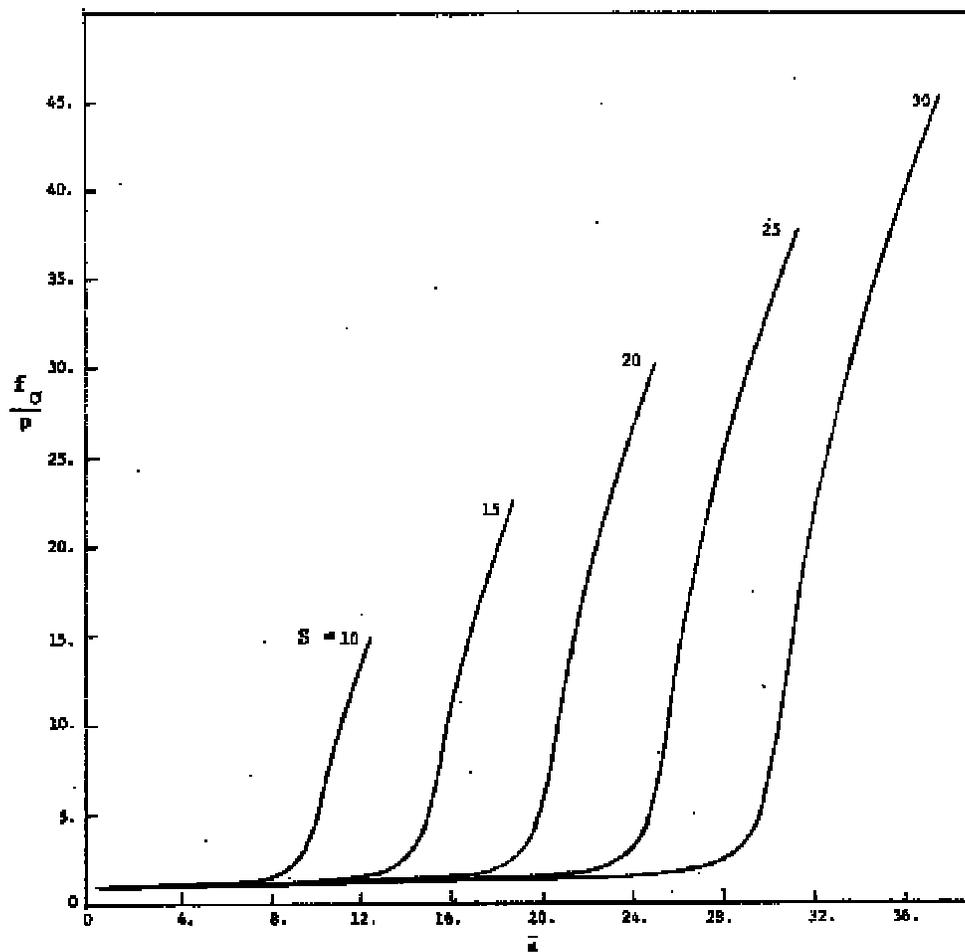


Figure 4.5.19. Normalized Panel Displacement Function (f_s/p) Due to Applied Stress vs. Normalized Crack Length (a/p) for Various Stringer Spacing ($s=S/p$)

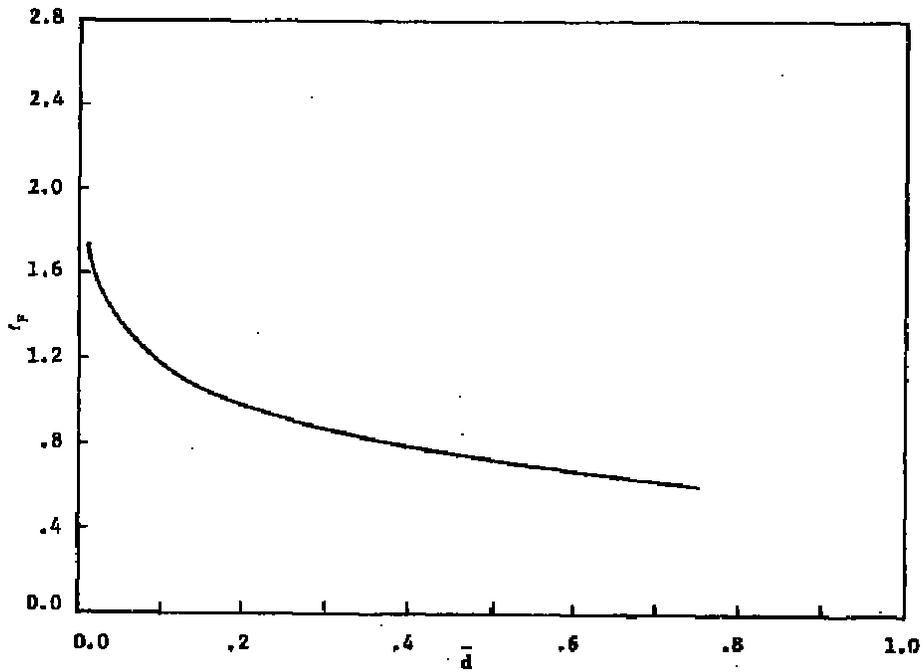


Figure 4.5.20. Panel Displacement Function Due to Fastener Force vs. Normalized Rivet Diameter (d/p) for All Stiffener Spacings

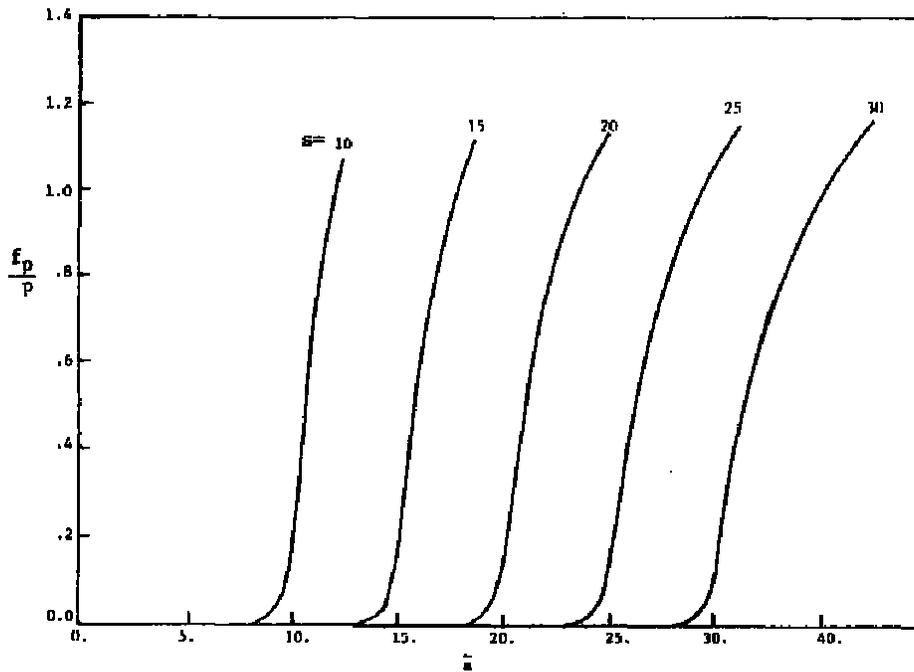


Figure 4.5.21. Normalized Panel Displacement Function (F_p/p) Due to Crack Distributed Pressure Along Crack vs. Normalized Crack Length (a/p) for Various Stringer Spacings ($s=s/p$)

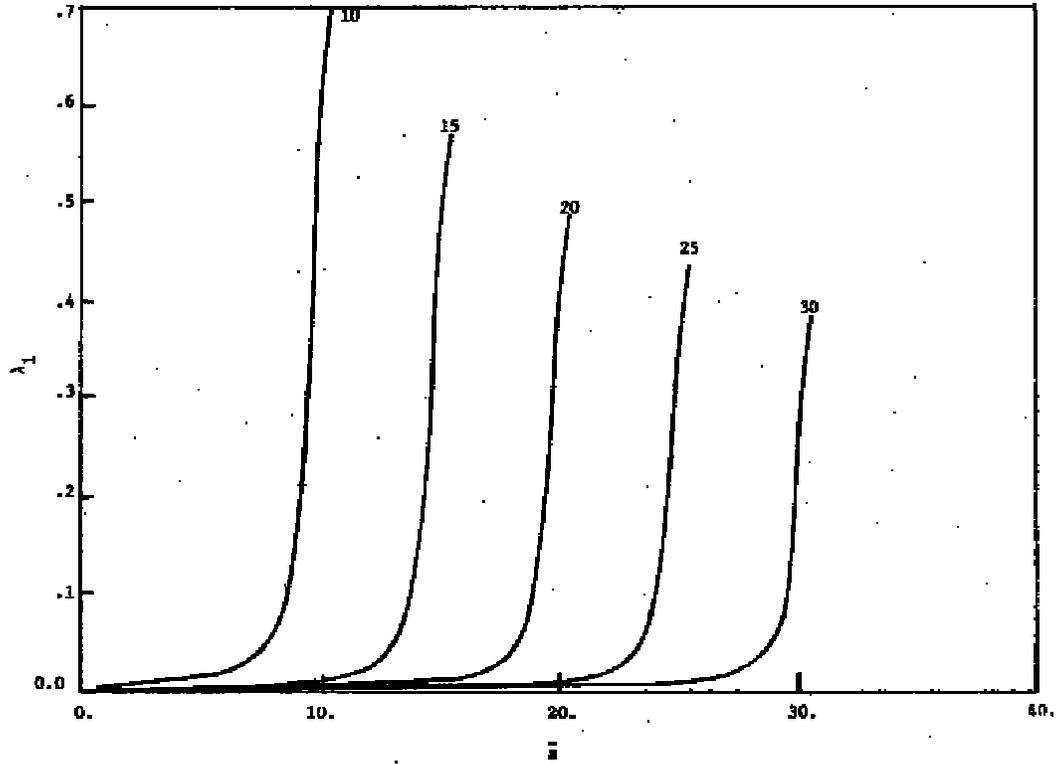


Figure 4.5.22. Stringer Displacement Function vs. Normalized Rivet Diameter (d/p) for Various Half-Stringer Widths

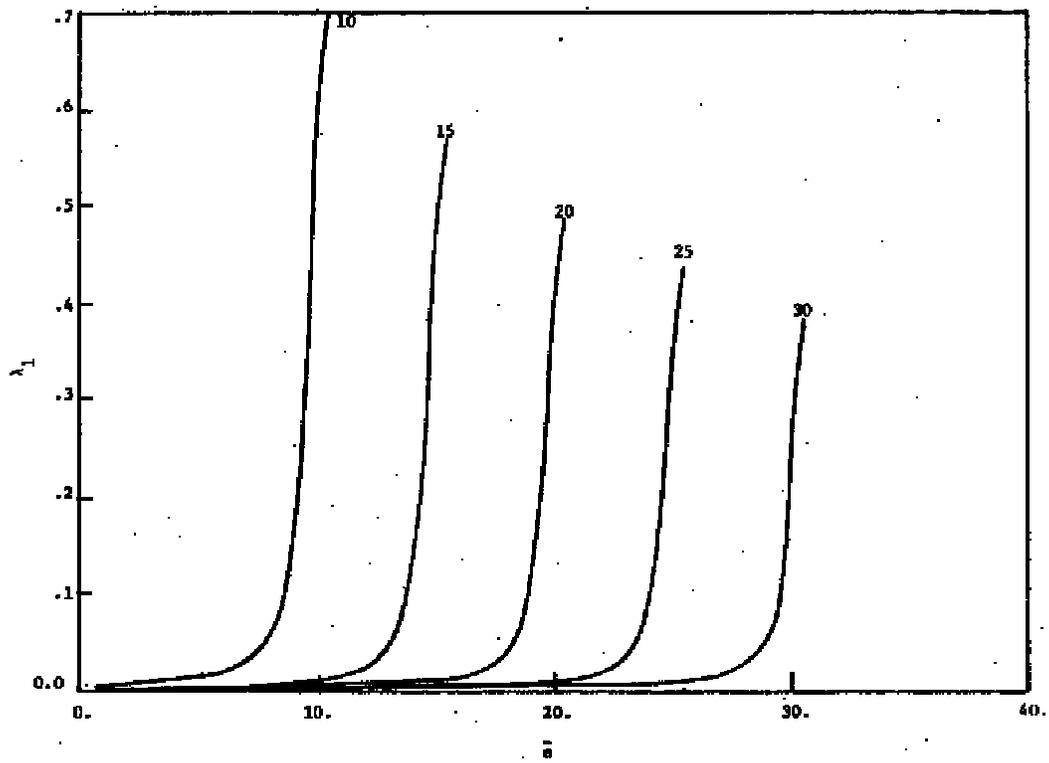
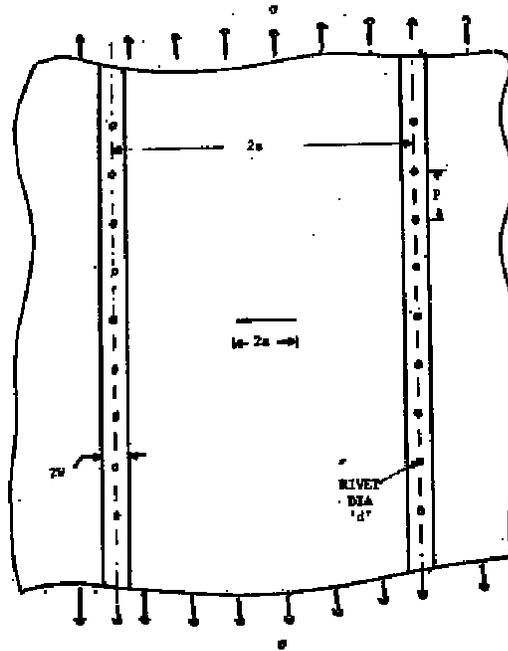


Figure 4.5.23. Parameter λ_1 Vs. Normalized Crack Length (a/p) for Various Normalized Stringer Spacings (s/p)

EXAMPLE 4.5.1 Residual Strength Analysis of Stiffened Panel

Determine the residual strength capabilities of a stiffened panel of 7075 aluminum with a central crack between the two stringers as shown in [Figure 4.5.24](#).

For a critical crack size ($2a$) of 4.0 inch, what is the fracture strength and for an operating stress of 20 ksi, what is the critical crack size?



given the following data:

Maximum operating stress: 20 Ksi
Stiffener spacing ($2s$): 40 inch
Stiffener width (w): 0.5 inch
Stiffener thickness (B_{st}): 0.063 inch
Panel thickness (B): 0.063 inch
Rivet pitch (p): 1 inch
Rivet diameter (d): 3/16 inch
Material: 7075-T6 Aluminum
Fracture toughness: 65 Ksi $\sqrt{\text{inch}}$
Ultimate Strength: 78 Ksi

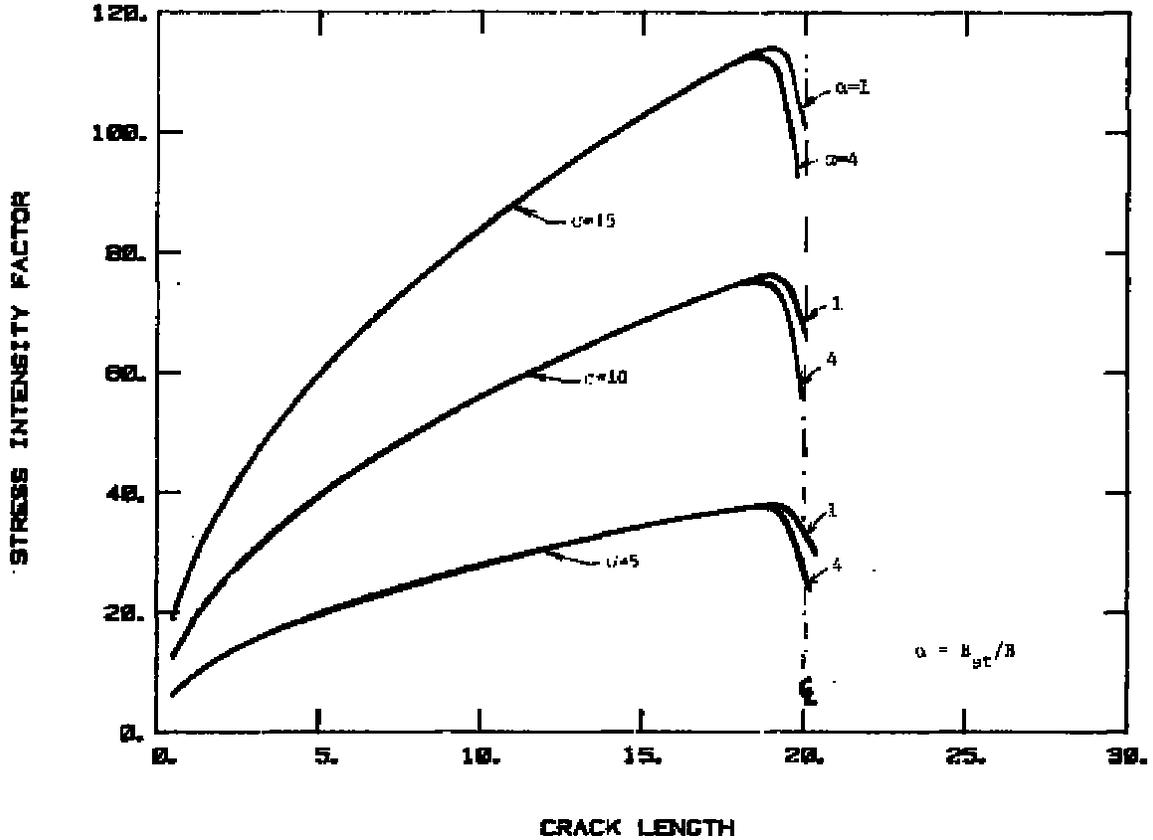
Structural Geometry and Material Properties for [Example 4.5.1](#)

SOLUTION:

The first step is to obtain the stress-intensity factor by means of Equation 4.5.21 that involves the parameters λ_1 and λ_2 . For various crack lengths, these two variables can be calculated using Equation 4.5.20. The calculations involve the values of f_σ , f_F , f_p , f_{st} and λ which are obtained from the plots for various values of \bar{a} for the given $\bar{s} = 20$ and $d/p = 3/16$. Knowing the values of λ_1 and λ_2 , the geometric parameter β can be estimated from Equation 4.5.22. It is then straightforward to obtain the K vs. a plot by substituting the sets of values of a and β in the stress-intensity factor equation ($K = \sigma\beta\sqrt{\pi a}$) for a particular value of the applied stress σ . The

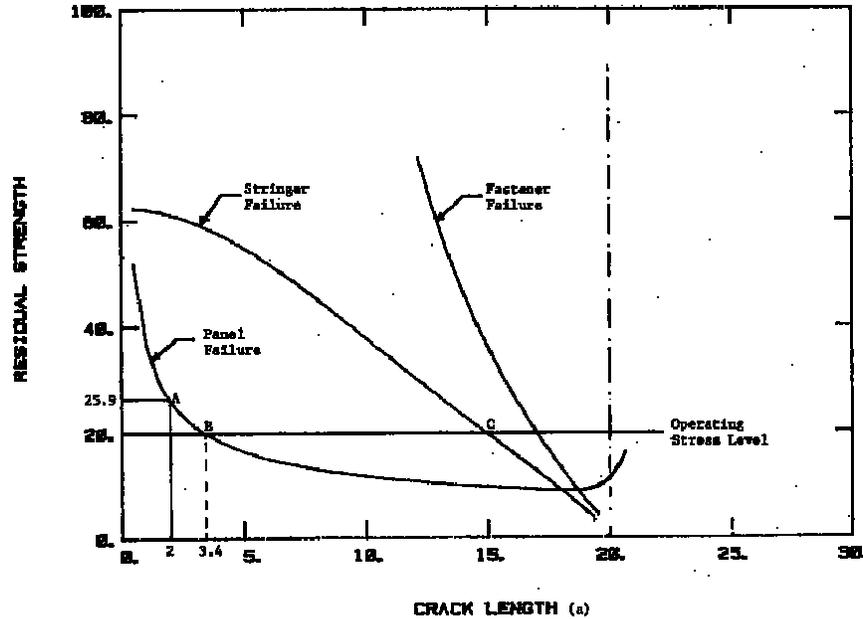
corresponding K vs. a plot is shown for $\sigma = 5, 10,$ and 15 ksi. This figure shows that the stress-intensity factor decreases rapidly when the crack approaches the stringer. The figure also shows the effect of stringer to panel thickness ratio on the stress-intensity factor.

STRESS INTENSITY FACTOR DIAG.



Stress Intensity Factor Diagram for Panel and Riveted Stringers

The next step is to apply a failure criterion to evaluate the fracture stresses, σ_{cr} , for various crack sizes. Assuming that the material exhibits negligible subcritical crack growth, the fracture toughness failure criterion ($K = K_{cr}$) based on the plane stress condition can then be applied. For $K = K_c$ in Equation 4.5.12, σ_f can be evaluated for a particular crack size and the corresponding β which was obtained through Equation 4.5.22. The residual strength diagram, i.e., the plot of σ_f vs. a_c for the given data ($K_c = 65 \text{ ksi} \sqrt{\text{in}}$), is shown in the following figure.



Residual Strength Diagram for Panel and Riveted Stringers (Light Stringers)

The residual strength curves of the fastener and stiffeners are obtained by combining the equations for fastener failure and the equations stringer failure. The corresponding equations are given by:

$$\sigma_f = \frac{\pi d^2 \tau_{ult}}{4\lambda} \text{ (Fastener)}$$

and

$$\sigma_f = \psi \frac{\sigma_{ult}}{\left(1 + \frac{\lambda}{A_s}\right)} \text{ (Stringer)}$$

where λ is a function of a , and the values of λ for various crack lengths can be obtained using the Equation 4.5.16. To obtain this Equation 4.5.24, note that the maximum stringer load (P_{max}) is the source of the fastener force ($F = \sigma\lambda$) and the remote stringer force (σA_s). The composite residual strength diagram as shown in the [figure](#) above contains the three failure curves corresponding to panel, stringer, and fastener. The stringer failure curve corresponds to $\alpha = 1$ (light stringer).

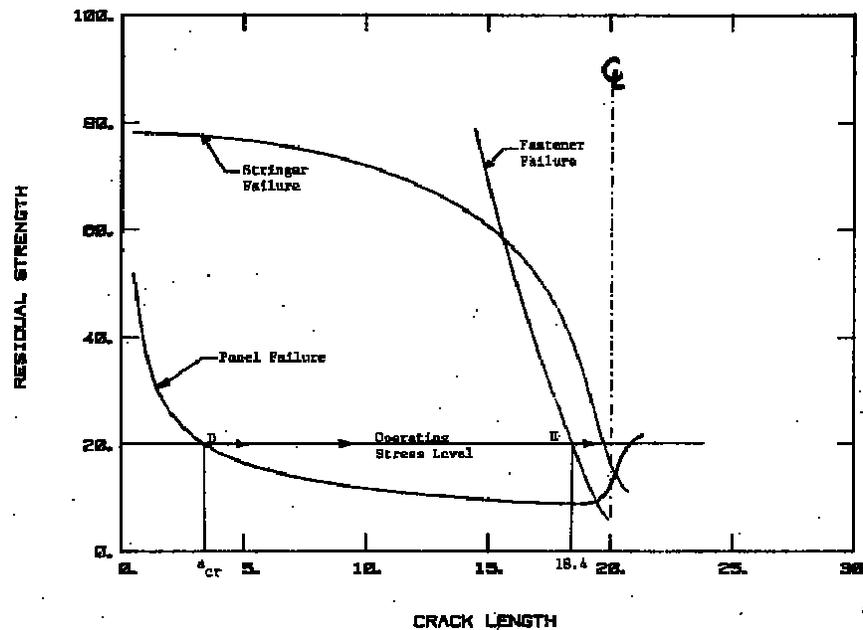
For the crack length given ($2a = 4$ inches), the corresponding residual strength is found from the figure for a half crack length (a) of 2 inches. Point A in this figure identifies the skin failure condition which occurs at a stress level of 25.9 ksi. For the operating stress level of 20 ksi, the panel can be effective without catastrophic failure for cracks with length less than the critical crack (a_{cr}) of 3.4 inch (note $2a_{cr} = 6.8$ inch). If the panel develops a crack less than a_{cr} , it will not fail by unstable crack growth. However, for any other crack size which is equal or

greater than the a_{cr} (3.4 inch), the residual strength level will fall below the operating stress level, leading to the rapid extension of the crack. Nevertheless, the structure has to be fully analyzed for its crack arrest capabilities when it develops cracks of length greater than a_{cr} .

Assume that the panel develops a crack of size a_{cr} . At point B in the figure, the crack extends rapidly. When the rapidly extending crack becomes 15 inches, the stress level in the stiffener (point C) reaches its critical value and the stiffener fails. Due to the stiffener failure, the stiffener becomes ineffective, leading to the total failure of the panel without any crack arrest possibilities.

In the next figure, the stiffener failure curve is plotted for a strong stiffener with $\alpha = 4$ (the stiffener thickness is "assumed" four times the panel thickness). If the panel develops a crack size a_{cr} , the crack will extend rapidly from point D to point E as shown in the next [figure](#). At point E, the fastener fails, leading to an ineffective stringer (loads are no longer transferred to the stringer). Thus, the failure of the panel is unavoidable and the unstable crack growth without effective crack arrest leads to the total failure of the structure.

RESIDUAL STRENGTH DIAGRAM



Residual Strength Diagram for Panel and Riveted Stringers (Heavy Stringers)

4.5.7 Tearing Failure Analysis

When the cracked thin sheet structure of high fracture toughness material is considered, the solutions based on linear elastic behavior for the calculation of residual strength are no longer valid due to the large scale yielding at the crack tip. For fail-safe structures with crack arrest capabilities, the residual strength analysis becomes complicated. However, using the R-curve based on $\sqrt{J_R}$ concept as the failure criterion Ratwani and Wilhem [1974] developed a step-by-step procedure for predicting the residual strength of built-up skin stringer structure composed of tough material exhibiting tearing type fractures.

The residual strength prediction procedure is briefly outlined here to show step-by-step, the required data and analysis. It should not be assumed that by reading this step-by-step procedure that the uninitiated can perform a residual strength prediction. It is strongly recommended that the details of the preceding subsections and Ratwani and Wilhem [1974] be examined prior to attempting a structural residual strength analysis based on the following ten procedural steps:

Step 1. Model the structure for finite-element analysis or use an existing finite-element modeling remembering –

- a. That structural idealizations are typically two-dimensional,
- b. That no out-of-plane bending is permitted,
- c. To use a proper fastener model (a flexible fastener model for riveted or bolted structure, or a shear spring model for bonded structure).
- d. To use material property data from skin and substructure of interest (i.e., E , E_{ty} , and F_m),
- e. To select the most critical crack location (normally highest stressed area),
- f. To take advantage of structural symmetry.

Step 2. Select one crack length ($2a$ or a) of interest (based on inspection capability or detailed damage tolerance requirement). Based on this “standard” crack length, five other crack lengths are selected for a Dugdale type elastic plastic analysis. These crack lengths should be selected such that crack length to stiffener spacing ($2a$) ratios vary between 0.15 to 1.1 remembering –

- a. That the greatest variation in J values will take place near reinforcements, and
- b. To select at least one crack size shorter than “standard”.

Step 3. With the finite-element model (from Step 1) and assumed crack lengths (from Step 2), perform an analysis assuming Dugdale type plastic zones for each crack size remembering –

- a. To select the first increment of plastic zone length at 0.2 inches and sufficient successive increments (normally 6) to reach Bueckner-Hayes calculated stresses up to 85 percent to F_{ty} .
- b. To make judicious selection of plastic zone increments so as to take advantage of overlapping a_e (effective crack length) (e.g., 3.2, 3.5, 4.2, 5.0 inches for a 3 inch physical crack and 4.2, 4.5, 5.0 inches, etc., for a 4 inch physical crack). If overlapping is done, those cases where the crack surfaces are loaded throughout

the crack length will be common for two or more physical crack sizes hence the computer programs need be run only once (e.g. 4.2 and 5.0 inches) thus reducing computer run times.

Step 4. From Step 3, obtain stresses in stiffeners for Dugdale analysis and elastic analysis. Plot stiffener stresses as function of applied stress.

Step 5. From the crack surface displacement data of Step 3, plot \sqrt{J} (obtained by Bueckner-Hayes approach) versus applied stress to F_{ty} ratio for each crack size.

Step 6. From Step 5, cross plot the data in the form of \sqrt{J} versus crack size (a) at specific values of applied stress to F_{ty} ratio.

Step 7. Employing the data of Step 4 and the “standard” crack size determine, gross panel stress to yield strength ratio, σ/F_{ty} at ultimate strength (F_u) for the stiffener material - assuming zero slow crack growth. This information will be used subsequently to determine if a skin or stiffener critical case is operative.

Step 8. Obtain crack growth resistance data for skin material (see Volume II of reference 26) remembering --

- a. To use thickness of interest (i.e., if the skin material is chemically milled, use the experimentally obtained R-curve for the same chemically milled material)
- b. Use proper crack orientation (LT, TL, or off angle) corresponding to anticipated direction structural cracking.

Step 9. Plot \sqrt{J} versus Δa_{PHY} curve as shown in [Figure 4.5.24](#) from the data obtained in Step 8.

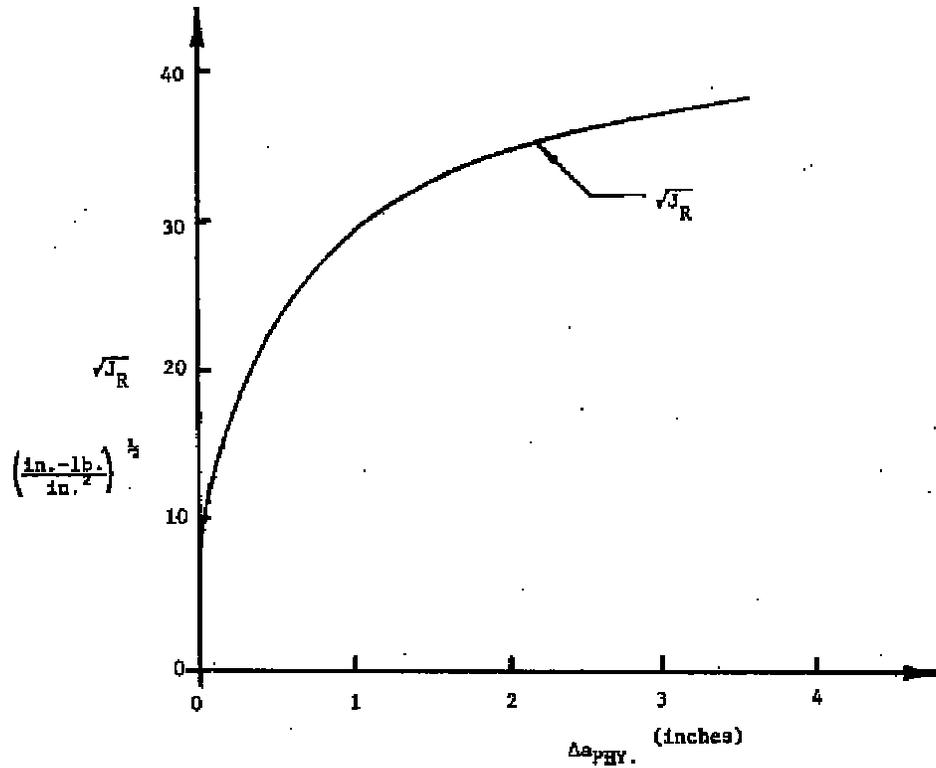


Figure 4.5.24. Square Root of J_I Resistance Curve

Step 10. Determine structural residual strength. On the \sqrt{J} versus crack size (a) plots obtained in Step 6 for the structure, overlay the $\sqrt{J_R}$ versus Δa_{PHY} material plot of Step 9 at the initial crack length of interest as shown in [Figure 4.5.25](#). Determine if –

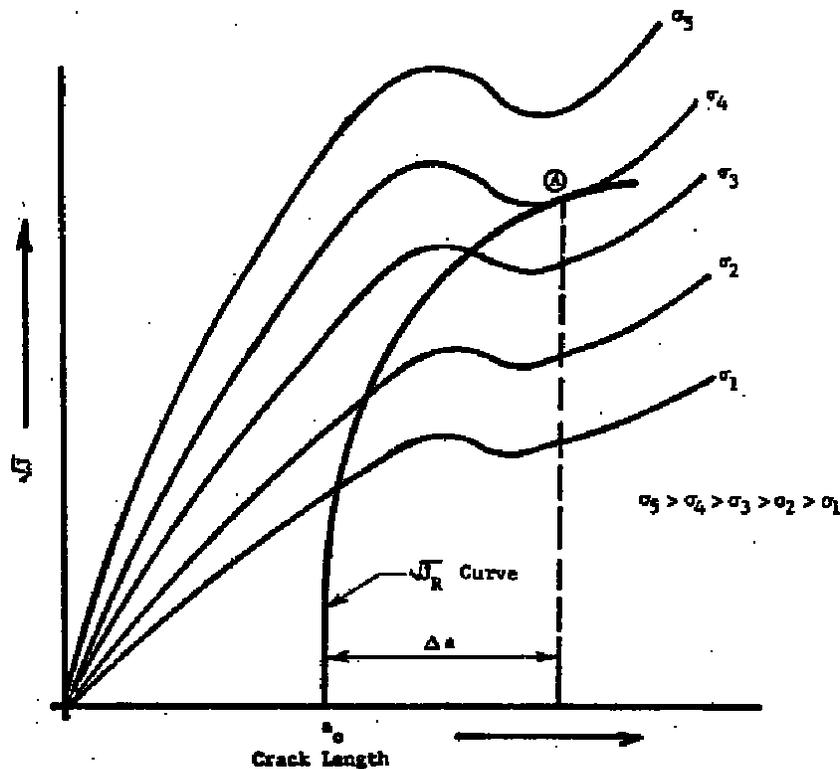


Figure 4.5.25. Failure Analysis Based on J critical Curve

At the gross panel stress obtained from Step 7, significant slow tear (≥ 0.25 inch) will occur as indicated from the intersection of the $\sqrt{J_R}$ versus Δa_{PHY} curve with the constant σ/F_{ty} curve at a stringer ultimate strength (see Step 7). Interpolation will probably be necessary between values of constant σ/F_{ty} . Then proceed as follows:

If significant slow tear occurs (> 0.25 inch) the structure can be considered to be skin critical (at that particular crack length). Tangency of $\sqrt{J_R}$ versus Δa_{PHY} and \sqrt{J} versus a_{PHY} at constant applied stress can be used to determine extent of slow tear and residual strength at failure as a percentage of F_{ty} .

If significant slow tear does not occur ($\Delta a_{PHY} < 0.25$ inch) the structure will normally be stiffener critical. To determine a conservative value of residual strength (for that crack length) use the Dugdale curve of Step 4 and stiffener ultimate strength.

4.5.8 Summary

The most important factor to consider in residual strength prediction of a cracked built-up structure is to decide whether the structure is skin or stiffener critical. Normally, a short crack length is likely to be a skin critical case and a long crack length a stiffener critical case. However, there is no clear cut demarcation between the two cases. Factors such as percentage stiffening, spacing of stringers, lands in the structure, and other structural details will influence the type of failure. Hence, a good technique is to determine the residual strength of a given structure based on both skin critical and stiffener critical cases. The minimum fracture stress of

the two will then represent the residual strength of the structure and should be considered to be the governing case.