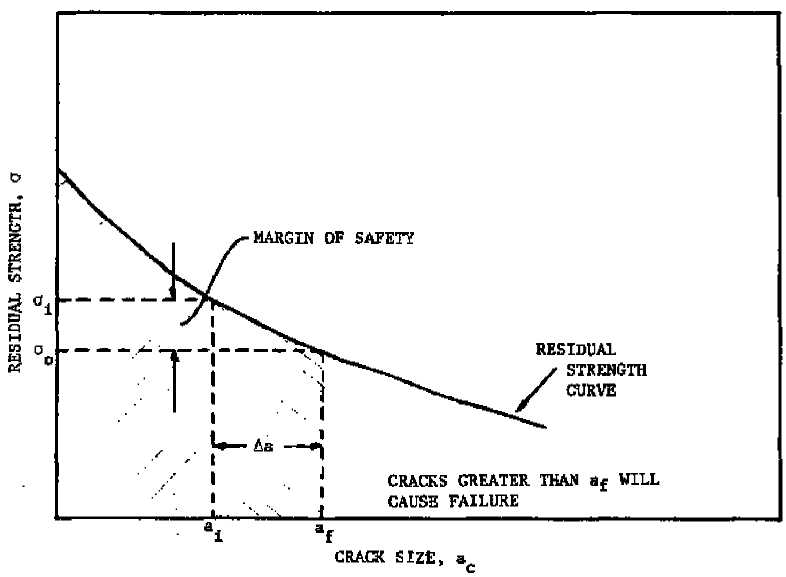
4.4 Single Load Path Structure

For a single load path structure, the only means to protect the safety is to prevent the damage growth from degrading the strength of the structure to less than the design limit load. This applies for all structures classified as slow crack growth, regardless of the type of construction (such as single load path or multiple load path). The residual strength capability of the structure depends mainly on the material's resistance to fracture.

4.4.1 Abrupt Fracture

For materials that exhibit abrupt failure, the start of slow crack extension will be followed immediately by the onset of rapid fracture. The residual strength capability then requires a strict evaluation of the initial flaw sizes in the structure. The allowable initial crack length necessary to maintain the required residual strength will be less than a_f; the design limit load must also be such that the stress level in the structure is less than σ_i , as shown in Figure 4.4.1. The residual strength diagram can be evaluated as described earlier through the plot of σ_f vs. a_c using the relationship $K = \sigma\beta\sqrt{\pi a}$ for the structural geometry of interest and also employing the failure criterion based on a critical fracture toughness value, K_{cr}. The margin of safety as shown in Figure 4.4.1 allows for undetected cracks or for subcritical crack growth such that the initial crack size will not become greater than a_f.



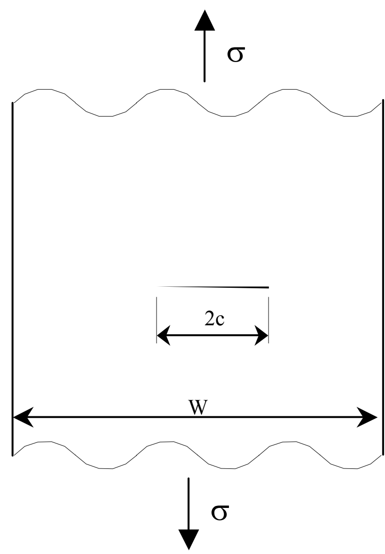
a_f = critical flaw size for the maximum operating load.
a_i = observed initial crack length.
Δa = available additional length. (allowance for undetected initial crack size and for subcritical crack growth).
σ_i = residual strength level for an initial crack a_i.
σ_o = maximum operating stress level.

Figure 4.4.1. Residual Strength Diagram Showing Defining Cracks and Residual Strength Parameters

The following example problem is presented to demonstrate the application of the steps in constructing the residual strength diagram and also to analyze the structure for its residual strength capabilities. This example demonstrates the basic concepts involved in the residual strength capabilities of a single load path structure.

EXAMPLE 4.4.1 <u>Residual Strength of Center Cracked Panel</u>

Develop the residual strength diagram for the cracked finite width panel shown here. The panel is 20 inches wide and 0.375 inches thick with a length of 60 inches. The yield strength (σ_y) for this material is 78 ksi and the fracture toughness (K_{Ic}) is 40 ksi √in. The inspection procedure is a viual inspection capable finding a crack (2*a*) 2 inches long.



SOLUTION:

For the center-cracked geometry configuration shown, the stress-intensity factor *K* expresses by the relationship (see Section 11.3):

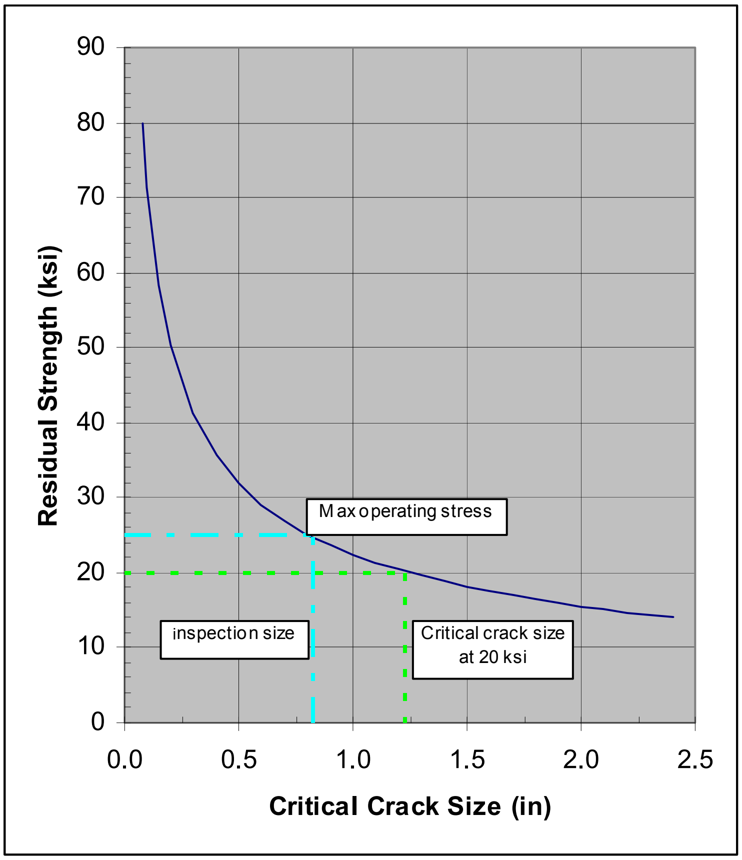
$$K = \sigma \sqrt{\pi a \sec\left(\frac{\pi a}{W}\right)}$$

Since we have an explicit expression for *K*, using the fracture toughness failure criterion (plane strain), the residual strength diagram can be obtained directly. The corresponding equation is

$$\sigma_f = K_{Ic} / \left(\sqrt{\pi a_c \, sec\left(\frac{\pi a_c}{W}\right)} \right)$$

where K_{Ic} = 40 ksi $\sqrt{in}$ and W = 20 inch are given as data and σ_f can be obtained for any selected crack length. The σ_f vs. a_c curve, which is the required residual strength diagram, can now be plotted.

The residual strength σ_f of the panel can be estimated from the equation that is described in the following diagram. From this figure, for the given operating stress level (20 ksi), the critical crack size a at which unstable crack extension would occur can be estimated as 1.2 inches. Thus, to avoid a fracture type failure of the panel, the structure should not develop a crack of this size. Assume that based on an established visual inspection schedule, the simple rectangular aluminum panel, uniformly loaded in tension as shown, could develop a 2.0 inch long, central through-the-thickness crack (normal to loading) before detection. This crack length is slightly smaller than the critical crack size (2a) of 2.4 (2 x 1.2 inch) under the operating conditions so that the margin of safety is small when this inspection process is employed.



Residual Strength Diagram Determining Critical Crack Size at 20 ksi Operating Level

To establish the required residual strength level to fit the inspection schedule, the designer must reduce the crack-tip stress-intensity factor for the same applied load. One method is to transfer portions of the load to a stiffening member. Another method is to reduce the operating load level below the failure level corresponding to the inspection crack size, although this is not always practiced.

4.4.2 Tearing Fracture

Materials with medium or high fracture toughness exhibit a type of subcritical crack extension behavior prior to reaching the maximum load carrying capacity of the structure. When a limited amount of yielding occurs in front of the crack tip, the initial extension of an existing crack in these materials will be slow and stable threshold values of the stress-intensity factor (K_{ONSET}).

To understand this behavior, consider an unreinforced, center-cracked panel. The stress-intensity factor (K) at the crack tip increases linearly with the value of the normal tensile stress component acting on the structure for a stationary crack. As the K level increases, some point (point A) will be reached at which the crack length will begin to extend as shown in Figure 4.4.2. The crack will extend gradually as the load continues to increase, until reaching the critical size at which the crack extension becomes unstable (point B in Figure 4.4.2). The point of crack initiation and instability are determined by the appropriate failure criteria.

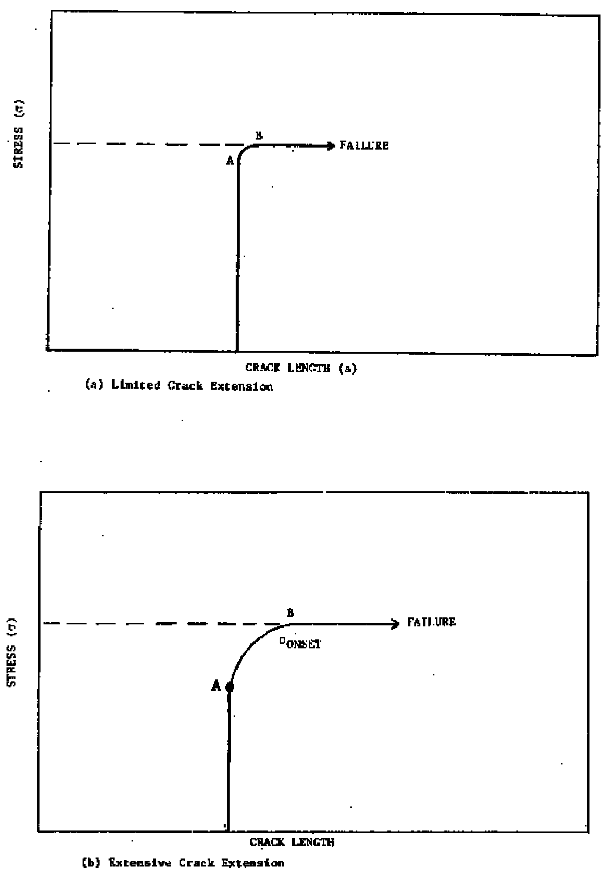


Figure 4.4.2. Diagrams Showing Onset of Unstable Crack Growth for Conditions of Limited or Extensive Crack Extension

When the subcritical growth of the crack, as shown in Figure 4.4.2a between the points A and B, is not significant, the fracture toughness criterion K_{CR} values can be used in the analysis. In this case, fracture is assumed to occur immediately after the start of crack extension as under abrupt failure conditions. However, for materials exhibiting substantial crack growth between points A and B as shown in Figure 4.4.2b, the crack resistance curve approach can be used in the residual strength analysis. The crack resistance (R) curve approach might be based on either K_R vs. Δa or $\sqrt{J_R}$ vs. a. The K_R vs. Δa curve is normally used when the fracture strength is associated with stress levels below net section yield conditions; in other words, when limited crack tip plasticity occurs prior to fracture. The $\sqrt{J_R}$ vs. a curve is used for those conditions where the fracture strength is expected to result in gross yielding.

In the calculation of residual strength when the cracked structure exhibits a tearing instability, one normally follows these steps:

1. Obtain $K_{eff} \left(= \beta\sqrt{\pi(a+r)}\right)$ values for the structure for various crack lengths and applied stresses using a suitable plastic zone model (e.g. Dugdale Model). Evaluation of the K values involves methods described in Section 11. Plot K versus a curves for various applied stresses as shown in Figure 4.4.3a.

2. Obtain the experimentally determined R-curve (K_R versus Δa) for the sheet material (Figure 4.4.3b).

3. Determine the point of instability from the K curves of the structure and the K_R curves of the material as shown schematically in Figure 4.4.3c.

4. Obtain different values for the fracture strength and the corresponding crack lengths from step 3 and plot these points to establish the failure strength (σ_f) crack length (a_c) curve. This provides the necessary residual strength diagram of the structure.

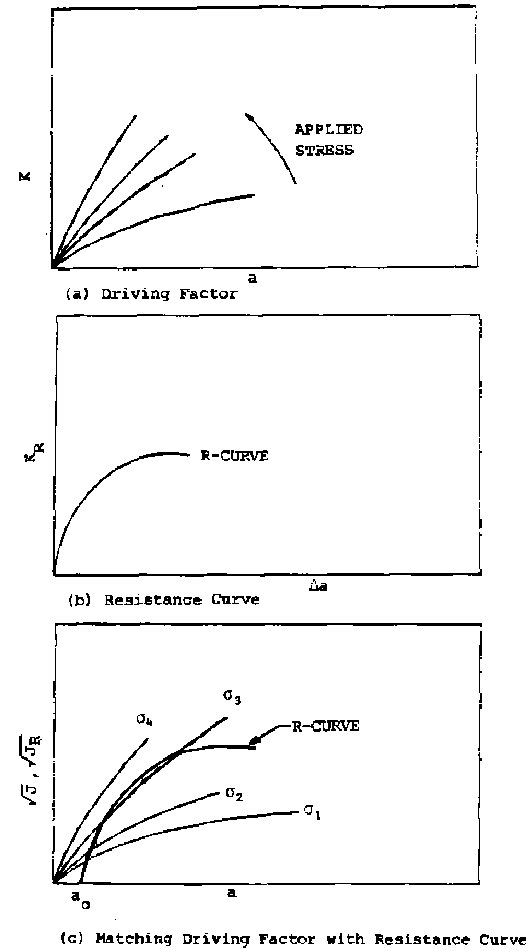
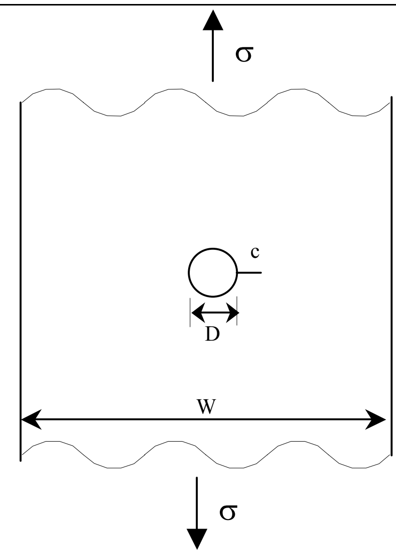


Figure 4.4.3. Steps Associated with Calculating Residual Strength of Cracked Structures with Tearing Fractures

The residual strength diagram for intermediate or high fracture toughness materials can be constructed by using either the K_R curve or the $\sqrt{J_R}$ method. To understand the use of the *R*-curve failure criteria in evaluating the residual strength, consider the following example in which failure criterion based on the K_R curve is applied.

EXAMPLE 4.4.2 Residual Strength of Tearing Radial Hole Crack

Construct the residual strength diagram for a large and relatively thin (0.063 in.) plate of 7075-T73 aluminum alloy having a through crack emanating (radially) from a hole with a diameter (*D*) equal to one inch, such as illustrated here. Assume the material inhibits a limited amount of crack tip yielding. Also calculate the crack length associated with a fracture strength associated with a crack length of 2.0 inch.



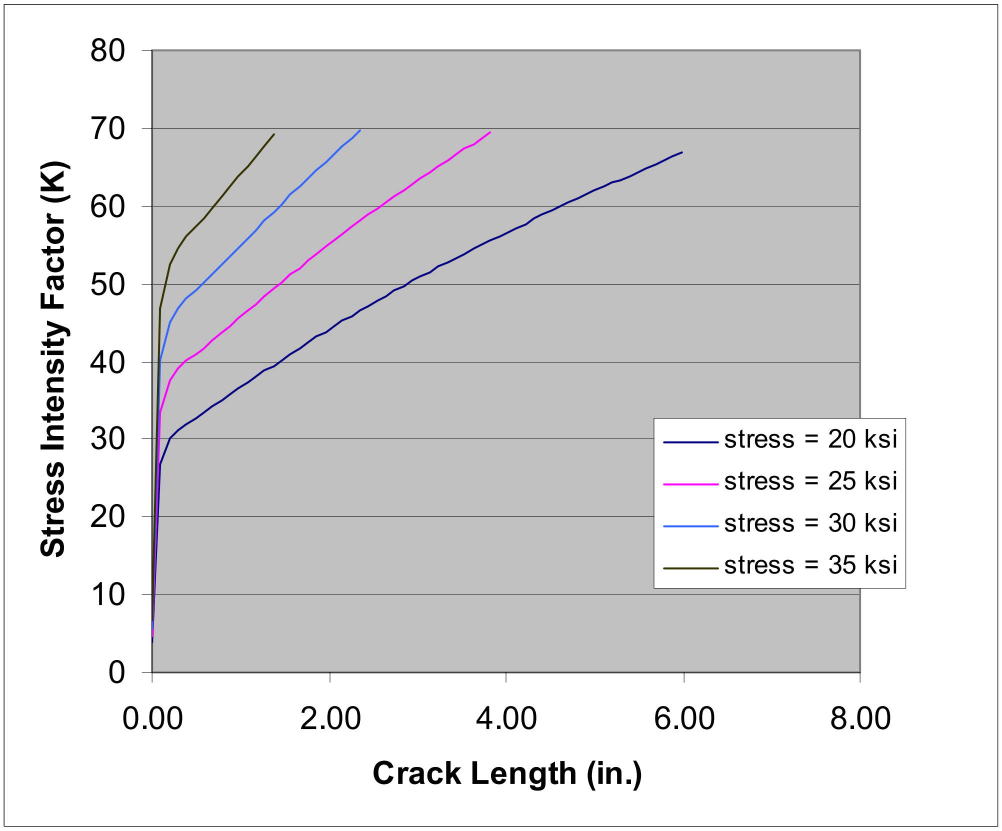
SOLUTION:

As the first step, the appropriate expression for the stress-intensity factor is obtained from Section 11:

$$K = \sigma\beta\sqrt{\pi c}$$

and β is given in Section 11.

The following figure describes the variation in stress-intensity factor with crack length and stress level.

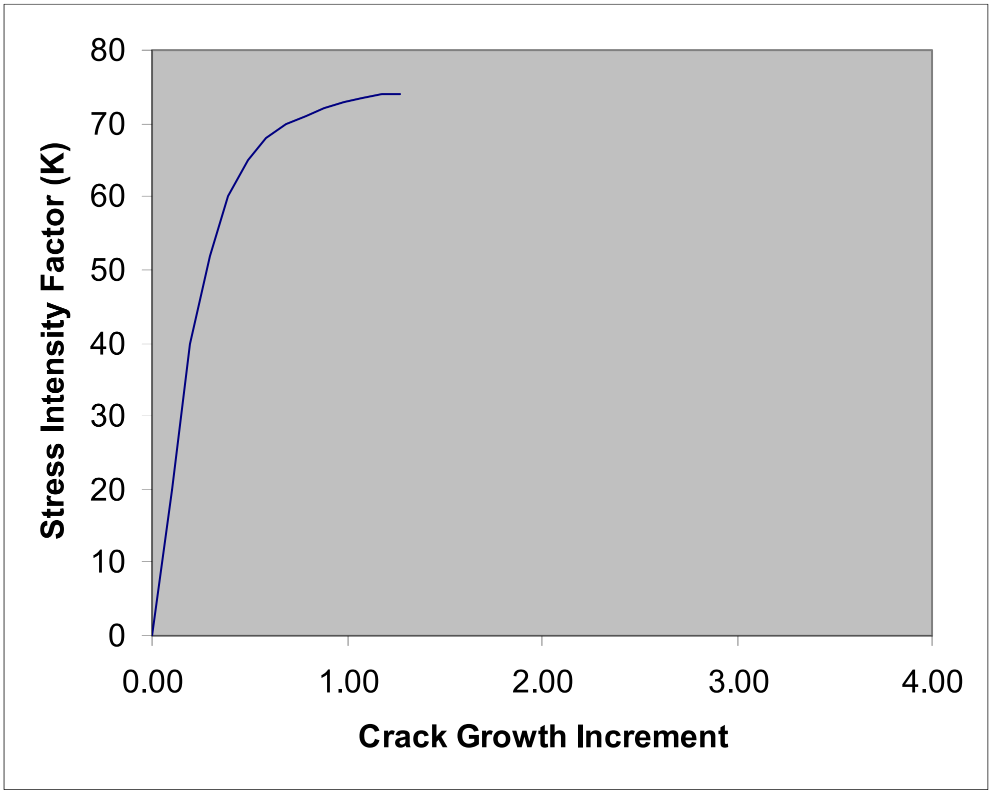


Stress-Intensity Factor Relationship for Various Values of Applied Stress

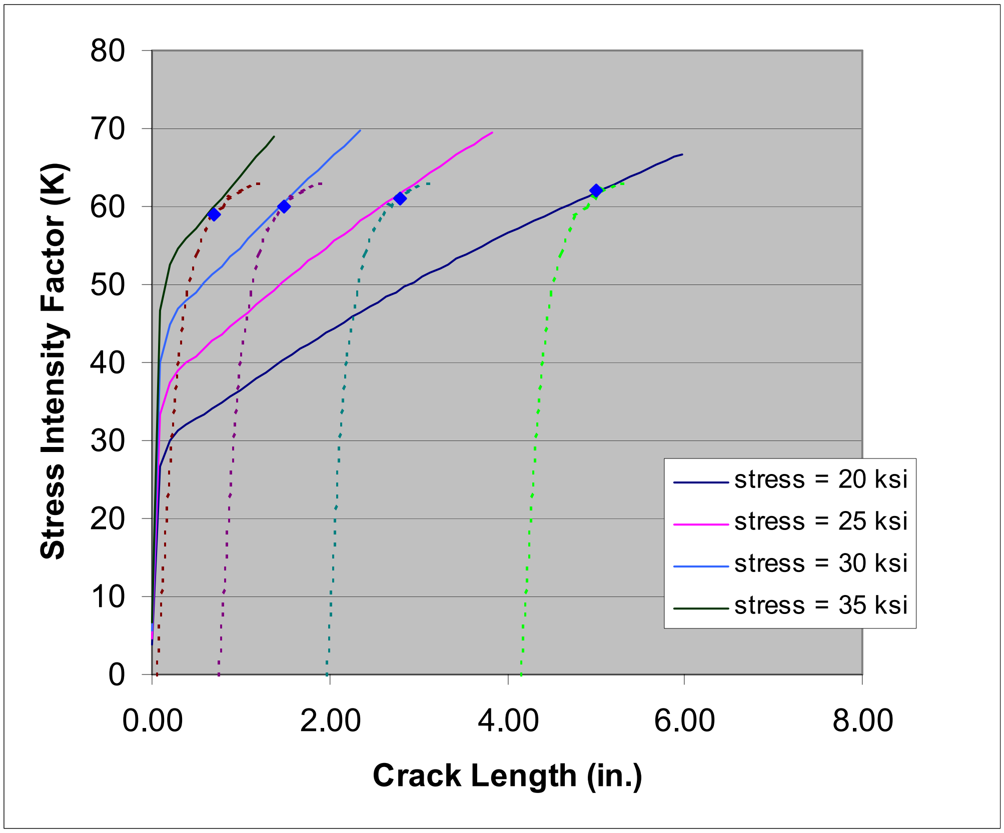
The next step is to consider the appropriate failure criterion. The given geometry is a thin sheet and the material exhibits limited crack tip yielding behavior. Therefore, the *R*-curve method based on K_R values can be applied to evaluate the fracture strength.

For the given 7075-T73 aluminum alloy material (0.063 inch thick), an experimentally obtained R-curve is shown here. By superposing the *R*-curve onto the plot obtained in step one, as explained in Section 4.2.1, the points where the *R*-curve is tangent to the *K*-curves are obtained.

At these points the failure criterion, i.e. K - K_R and $\frac{\partial K}{\partial a} = \frac{\partial K_R}{\partial a}$, is satisfied. The corresponding stress σ_c is the critical (fracture) stress at which the initiation of rapid fracture will occur. From a diagram like this, we can obtain the critical initial sizes of the crack and the respective fracture stresses.

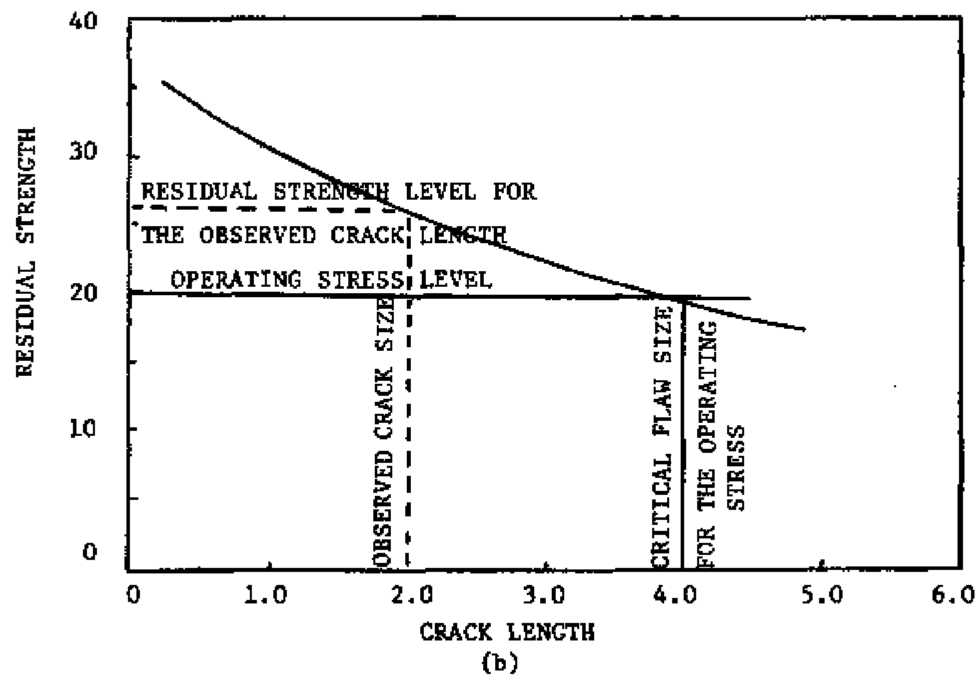


Resistance Curve for 7075-T73 Aluminum for a Thickness of 0.063 Inches



Matching the *R*-Curve and Stress-Intensity Factor Curves

The final step is to plot the σ_f vs. a_c curve. The required residual strength diagram is shown next for the 7075-T73 aluminum plate with a crack emanating radially from a hole. It can be seen from this figure that the critical crack size for a 20 ksi operating stress level is equal to 4.0 inches. As can also be seen from the figure, for an observed crack of 2.0 inches, the residual strength available is 27 ksi.



Residual Strength Diagram Obtained for Structure in Example 4.4.4