

## 2.4 Life Prediction Methodology

Currently, within the Air Force, airframe life predictions are based on a crack growth damage integration package that uses a data base and analysis to interrelate the following six elements:

- a) The initial flaw distribution which accounts for size variations and location of cracks in a given structure;
- b) aircraft usage describing the load spectra data base;
- c) constant amplitude crack growth rate material properties accounting for stress ratio and environmental effects;
- d) crack tip stress intensity factor analyses which account for crack size, shape, and structural interactions;
- e) damage integrator model which assigns a level of crack growth for each applied stress application and accounts for load history interactions; and
- f) the fracture or life limiting criterion which establishes the end point of the life calculation.

Prior to describing each of the above itemized elements in separate subsections, the damage integrating equation will be introduced to show how the various elements interact. As expressed in a numerical form, the damage integrating equation is

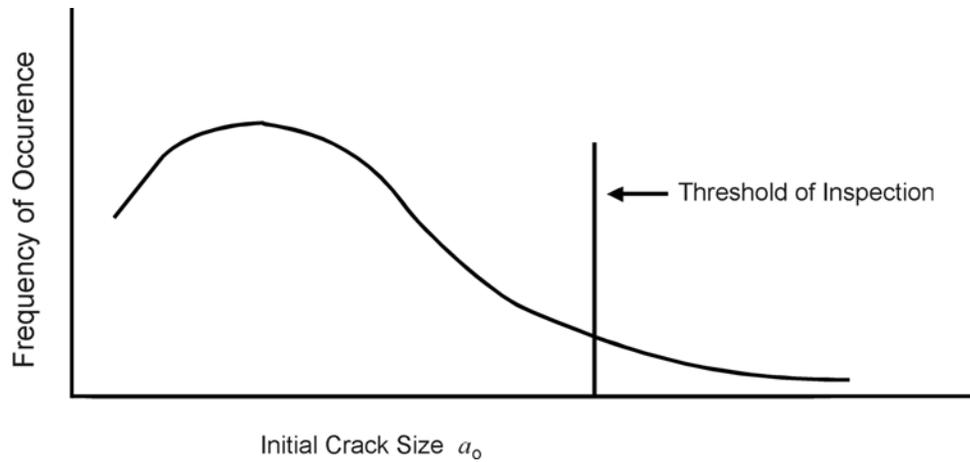
$$a_{cr} = a_o + \sum_{j=1}^{t_f} \Delta a_j \quad (2.4.1)$$

where  $\Delta a_j$  is the growth increment associated with the  $j^{th}$  time increment. The purpose of Equation 2.4.1 is to determine the life  $t_f$ . The various elements affect the quantities in Equation 2.4.1 in the following manner:

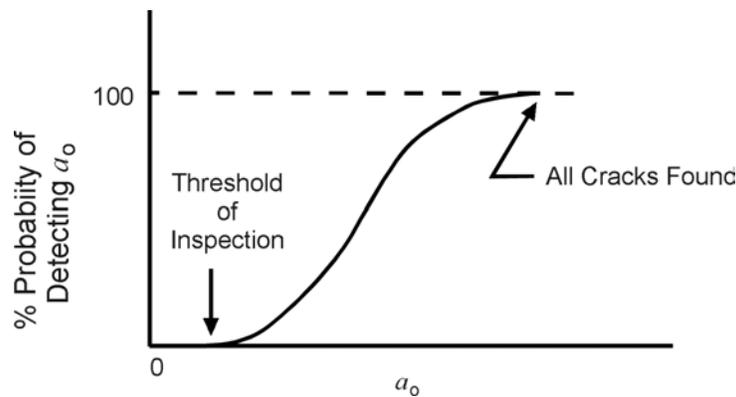
1.  $a_{cr}$  is determined interrelating elements b, d, and f.
2.  $a_o$  is determined using element a.
3.  $\Delta a_j$  is determined by interrelating elements a, b, c, d, and e.

### 2.4.1 Initial Flaw Distribution

A measure of initial quality in a component of service hardware is given by the distribution of initial crack sizes as illustrated in [Figure 2.4.1](#). For predictions of safety limits, the initial cracks larger than the nondestructive inspection (NDI) detectability limit are of principal concern. Current specifications detail NDI limits and require verification/certification of contractor capability to detect cracks smaller than the specified NDI limits. Normally, such certification is demonstrated with curves of the type shown in [Figure 2.4.2](#). The program of certification for a contractor's quality control inspector/inspection techniques allows the USAF to assess the probability and confidence limits associated with detecting a given crack. Section 3 will present a state-of-the-art summary of the technology and equipment that supports the establishment of initial flaws via nondestructive tools.

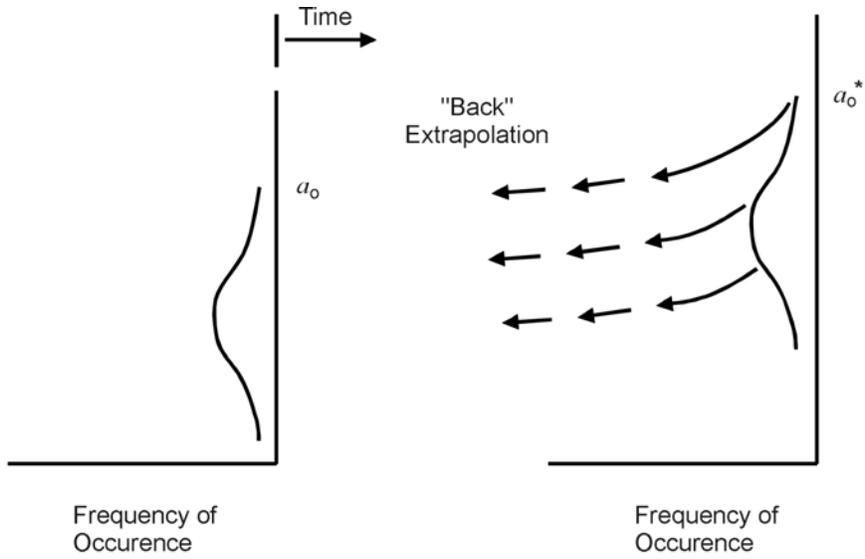


**Figure 2.4.1.** Distribution of Initial Crack Size for a Given Type of Crack (e.g., Radial Cracks Growing from Fastener Holes)



**Figure 2.4.2.** Certification of NDI Capability

Results generated by the F-4 Independent Review Team (IRT) provided a method of characterizing the initial flaw population (apparent initial quality) based on full-scale fatigue test-induced cracking behavior [Lozano, et al., 1974]. Given the measurable flaw distribution in a structure at some time subsequent to test startup, the initial flaw population can be backtracked by analysis. The “back” extrapolation of the flaw population is conducted using the damage integration package. The process is schematically illustrated in [Figure 2.4.3](#). Subsequently, the initial flaw distribution established as illustrated in [Figure 2.4.3](#) can be used to estimate influence of load factors, mission profiles, and usage changes on the life of service hardware. The F-4 IRT study also provided an evaluation of statistical methods for describing the large crack length extremes for initial flaw distributions established in this manner. The resulting distribution of F-4 initial cracks is shown in [Figure 2.4.4](#) [Lozano, et al., 1974; Pinchert, 1976].



a) Initial Flaw Distribution

b) Flaw Distribution Found After Fatigue Test

Figure 2.4.3. Determining Initial Quality by Back Calculation

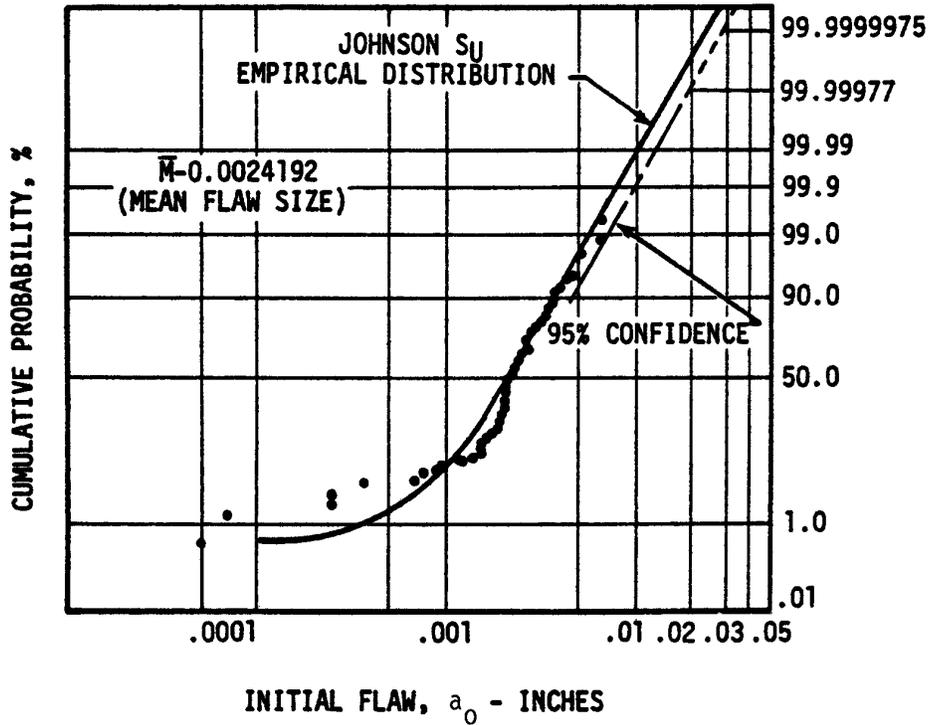
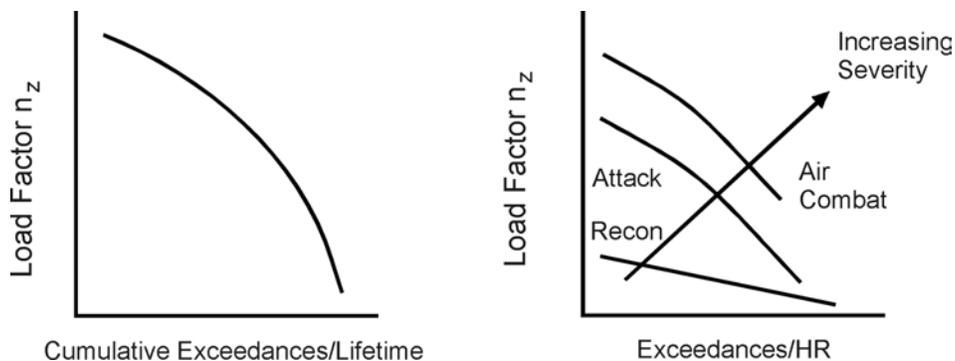


Figure 2.4.4. Initial Flaw Distribution for F-4 Based on Back Calculation

## 2.4.2 Usage

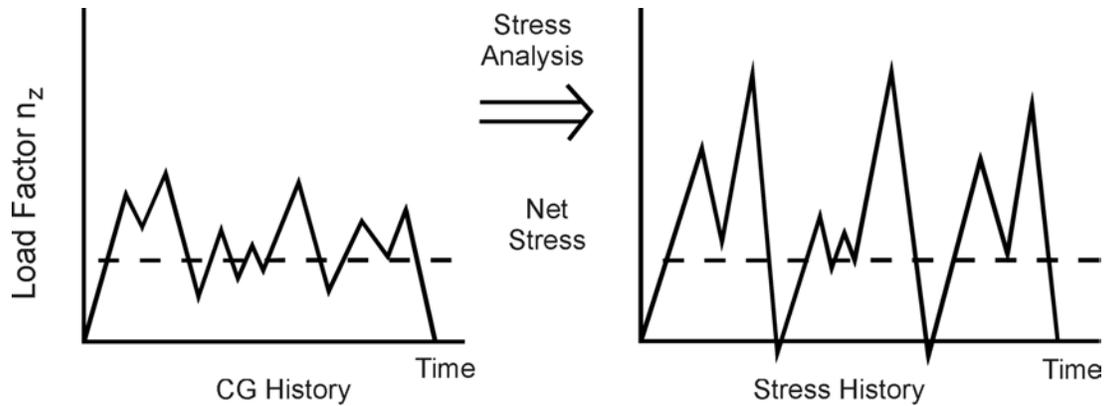
The sum of the load levels that a structure is expected to experience is determined by a projection of the amount of usage expected over the life in the various possible missions; e.g., hours in training, air-to-air combat, reconnaissance, weapons delivery, etc. The mission mix includes the relative amounts of time spent in each mission. The most basic information needed is the load factor exceedances at the center of gravity (CG) of the aircraft. This information is illustrated in [Figure 2.4.5](#). For new designs, this data is derived from actual measured exceedances from operational aircraft flying similar missions. The USAF specifications contain such data. The Air Force Guidelines Handbook for developing Load/Environmental Design Spectra [Giessler, et al., 1981] summarizes the techniques that are currently being utilized to develop the loading and environmental spectra based on these data for various types of structures.



**Figure 2.4.5.** Typical Load Factor Exceedance Information Indicating Usage

The specific sequence of loads applied to the structure is necessary to the crack growth damage accumulation analysis. Current practice is to simulate the overall life on a flight-by-flight basis. Each flight in the design, analysis, or test load spectrum consists of a series of cycles that combine the deterministic and probabilistic events describing the type of mission. The deterministic events include takeoff and landing, and certain basic maneuver loads during each flight. Probabilistic events such as gusts or rough field taxiing occur periodically. Although it is possible to estimate the number of times these events occur, their position in the load sequences is determined in a probabilistic manner.

In developing the load spectrum for crack growth damage analysis, it is necessary to determine the stress history for each critical area on the airframe. This is accomplished by determining the relationship between the load history derived above and the stress response. [Figure 2.4.6](#) schematically illustrates the load factor to stress history transformation.

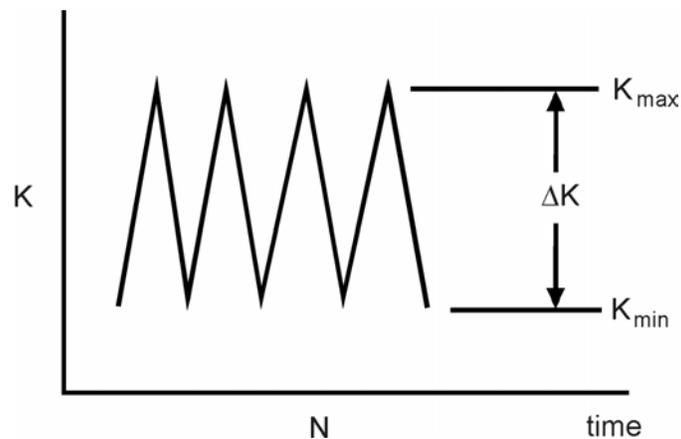


**Figure 2.4.6.** Load Factor to Stress History Transformation

Differences in crack growth resulting from mission mix can be significant. A fighter aircraft that is used primarily for air-combat or air-combat training typically accumulates more damage than one that is used for the same number of hours on a reconnaissance-type mission.

### 2.4.3 Material Properties

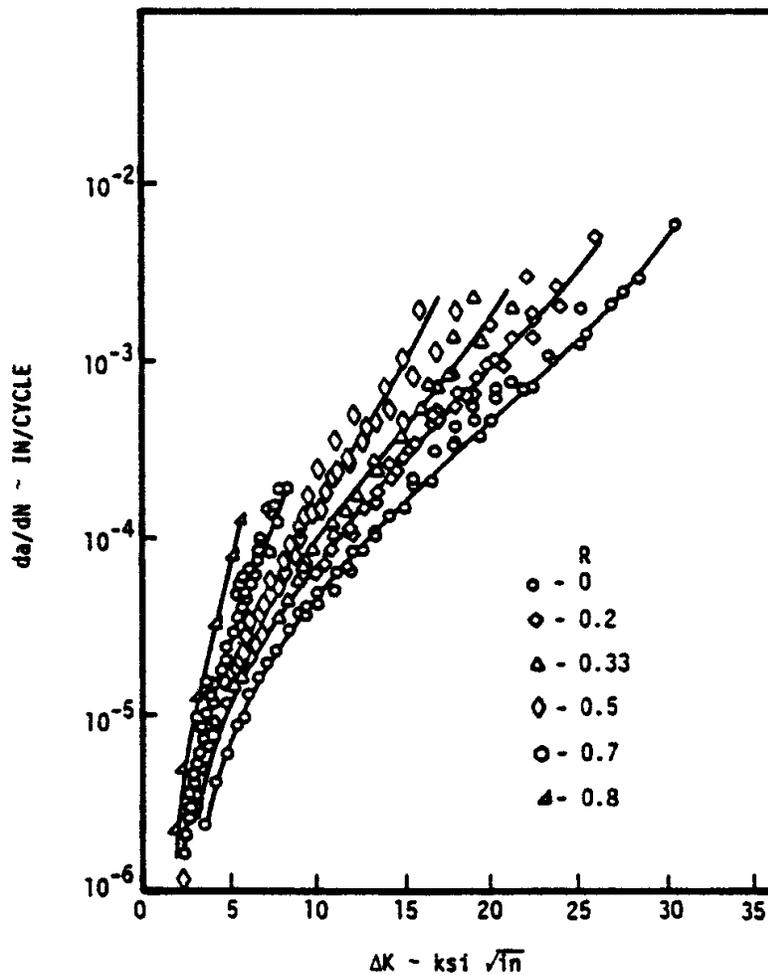
The material properties enter the damage integration package in the form of constant amplitude crack growth rate data. Crack growth data are generated in the laboratory under constant cyclic loading on simple specimens with accepted characterizing stress intensity factors. Crack growth rate data are developed and correlated on the basis of growth rate ( $da/dN$ ) as a function of stress intensity factor range,  $\Delta K$ , ( $\Delta K = K_{max} - K_{min}$ ), as defined in [Figure 2.4.7](#). The ASTM defines  $K_{min} = 0$  and thus  $\Delta K = K_{max}$  whenever  $R < 0$  ( $R = \sigma_{min}/\sigma_{max}$ ); see Section 5.1 for additional discussion.



**Figure 2.4.7.** Stress-Intensity Factors – Cyclic Loading

For a given  $\Delta K$ , the crack growth rate increases with increasing stress ratio,  $R$  for  $R > 0$ . Hence, the constant amplitude crack growth rate properties for a given material or alloy consist of a family of curves as illustrated in [Figure 2.4.8](#). The crack mechanics approach described in Section 2.2.1 considers that for a given  $\Delta K$ ,  $R$  combination, there is a  $da/dN$  that is independent

of geometry. Thus, the damage integration package has available a growth rate for each  $\Delta K$  determined for the given crack configuration and loading.



**Figure 2.4.8.** Constant Amplitude Crack Growth Rate Data for 7075-T6 Aluminum.

When necessary, thermal or chemical environment and time (frequency of loading) effects are also included in the crack growth rate data generated for use with the damage integration package.

Section 7 presents a summary of the currently available procedures and techniques which are used to establish crack growth rate data.

#### 2.4.4 Crack Tip Stress Intensity Factor Analysis

The crack tip stress intensity factor ( $K$ ) interrelates the crack geometry, the structural geometry, and the load on the structure with the local stresses in the region of the crack tip. The stress intensity factor takes the form

$$K = \beta\sigma\sqrt{\pi a} \tag{2.4.2}$$

where

$\beta$  - geometric term for structural configuration, can be a function of crack length

$\sigma$  - stress applied to the structure

$a$  - crack length

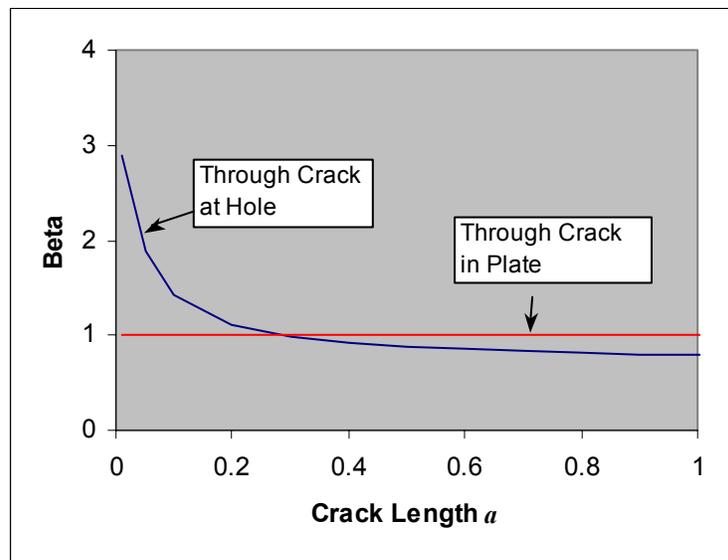
It can be seen that any number of combinations of the parameters  $\beta$ ,  $a$ , and  $\sigma$  can give rise to the same  $K$ . The crack growth analysis rests on the experimentally verified proposition that a given  $K$  gives rise to a certain crack growth rate, regardless of the way in which the parameters were combined to generate that  $K$ .

A considerable body of data exists which defines experimental and mathematical solutions for stress intensity factors for various structural configurations. A review of the procedures for obtaining stress intensity factors is covered, and the  $K$  solutions for a number of practical structural geometries are presented in Section 11.

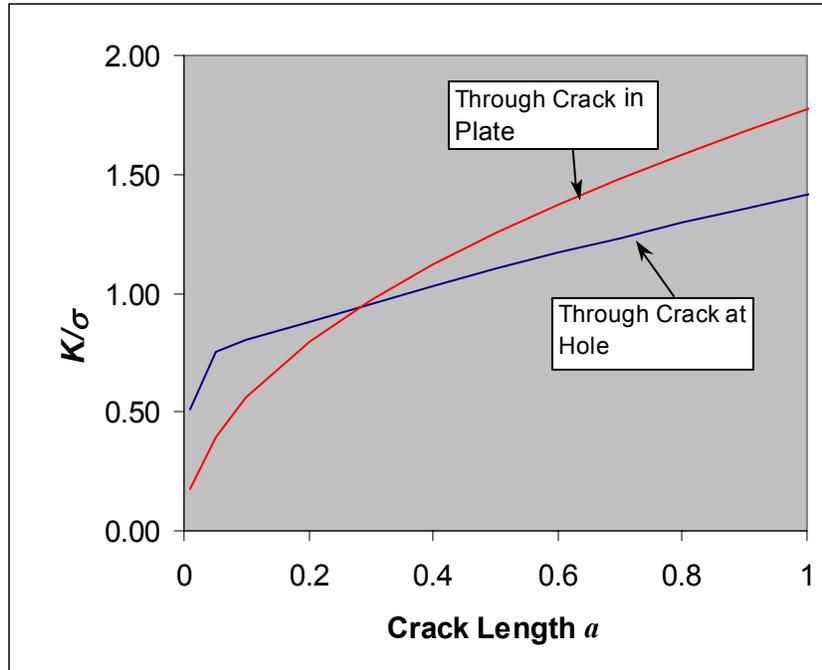
Since stress enters Equation 2.4.2 in a linear sense it is appropriate to express the geometrical part of the stress intensity factor by using the stress intensity factor coefficient,  $K/\sigma$ . [Figure 2.4.9](#) illustrates two typical solutions expressed in this manner. For a through-the-thickness crack in a plate of infinite extent, the value of  $\beta$  is unity and  $K$  becomes

$$K = \sigma\sqrt{\pi a} \quad (2.4.2a)$$

Equation 2.4.2a provides one way of normalizing more complex  $K$  solutions in terms of the infinite plate solution. [Figure 2.4.10](#) depicts a typical solution of this type.

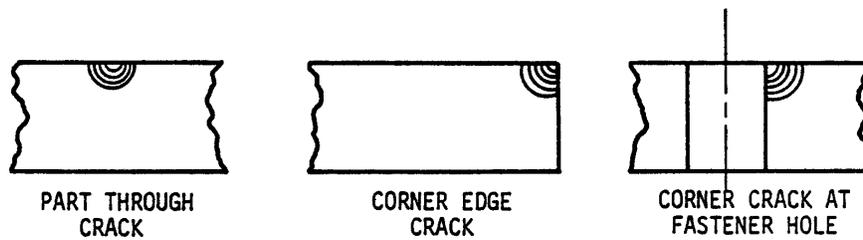


**Figure 2.4.9.** Stress-Intensity-Factor Coefficients Showing Influence of Hole on  $K$



**Figure 2.4.10.** Influence of Hole on Geometric Correction Factor

Through-the-thickness cracks are handled quite well analytically. However, for corner cracks and semi-elliptical part-through cracks, such as illustrated in [Figure 2.4.11](#),  $K$  varies from point to point around the crack perimeter. This variation allows the crack shape to change as it grows, which leads to a complex three-dimensional problem. The determination of  $\beta$  and  $K/\sigma$  for these complex cases have received a substantial amount of attention (see Section 11).



**Figure 2.4.11.** Complex Crack Geometries

### 2.4.5 Damage Integration Models

Rewriting Equation 2.4.1 such that the integration is conducted between the initial crack length ( $a_o$ ) and any intermediate crack length ( $a_K$ ) between  $a_o$  and the critical crack length results in

$$a_K = a_o + \sum_{j=1}^t \Delta a_j \quad (2.4.3)$$

where  $t(N)$  is the elapsed time (number of load cycles) corresponding to growing the crack  $a_o$  to the intermediate crack length  $a_K$ . The next cycle of the applied stress (the  $N + 1$  cycle) induces a

crack length growth increment  $\Delta a_{N+1}$ . The damage integration model provides the analysis capability to determine this crack length growth increment. The growth increment  $\Delta a_{N+1}$  is equated to the constant amplitude crack growth rate, which in turn is determined from a function of stress intensity factor range ( $\Delta K$ ) and stress ratio ( $R$ ), i.e.,

$$\Delta a_{N+1} = \left. \frac{da}{dN} \right|_{N+1} = f(\Delta K_{N+1}, R_{N+1}) \quad (2.4.4)$$

The stress intensity factor range and stress ratio in Equation 2.4.4 are determined by using the maximum and minimum stresses in the  $N+1$  cycle of the given stress history and evaluating the stress intensity factor coefficients associated with the given structural geometry at the crack length  $a_K$ . Subsequent to the direct calculation of the two crack tip parameters  $\Delta K$  and  $R$ , and prior to their insertion in Equation 2.4.4,  $\Delta K$  and  $R$  are modified to account for the effect of prior load history using retardation models. Retardation models account for high-to-low load interaction effects, i.e., the phenomena whereby the growth of a crack is slowed by application of a high load in the spectrum. Failure to account for high-to-low load interaction via a retardation model leads to conservative ( $\sim 2$  to 5 times shorter) life.

There are numerous functional forms of Equation 2.4.4 and numerous models describing retardation. The following list describe the general scheme of the crack growth calculation.

- Step 1** - Knowing crack length  $a_K$ , determines the stress intensity factor coefficient,  $K/\sigma$ .
- Step 2** - For the given stress cycle,  $\Delta\sigma$ , and the coefficient  $K/\sigma$ , determine the stress intensity factor cycle,  $\Delta K$ , and stress ratio  $R$ .
- Step 3** - Utilizing the retardation model, modify the stress-intensity cycle  $\Delta K$  and  $R$  to account for previous load history.
- Step 4** - Determine the growth rate for the stress-intensity factor cycle to establish the crack growth increment.

Section 5 provides a current state-of-the-art summary of the procedures and techniques that are used in damage integration models.

## 2.4.6 Failure Criteria

The interrelationship between critical crack length, loading, and residual strength of a structure was first discussed in Section 2.2 using Figure 2.2.3. Based on the information presented in Section 2.3.1, the residual strength ( $\sigma_{res}$ ), the load-carrying capacity of the cracked structure, can be shown to monotonically\* decrease with increasing crack length in the following manner:

$$\sigma_{res} = K_c/f(a) \quad (2.4.5)$$

where

$K_c$  = the material resistance to fracture, termed fracture toughness, and

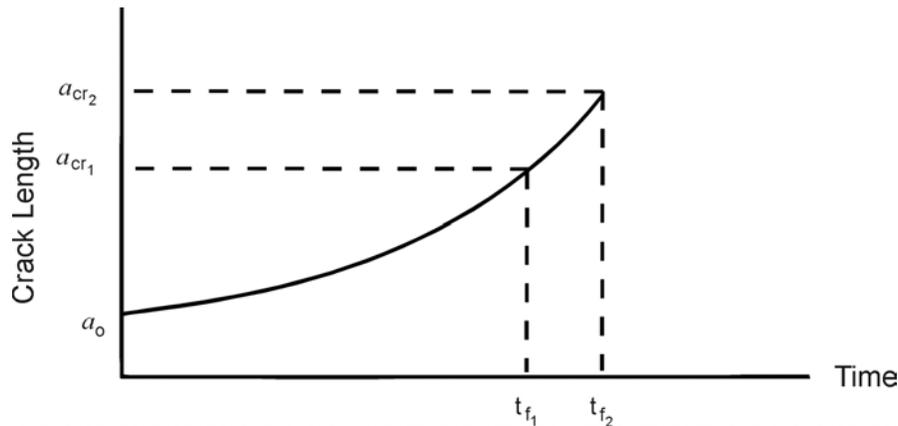
$f(a) = \beta(a)\sqrt{\pi a}$ , the structural property, termed the stress intensity factor coefficient.

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\* monotonic implies that the rate of change does not change sign.

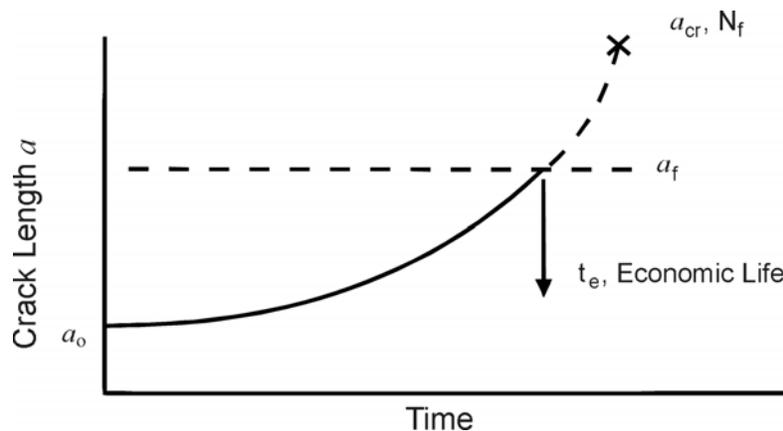
When the residual strength decays to the level of the maximum stress in the service load history, fracture of the structure occurs. The crack length associated with fracture (i.e.,  $a_{cr}$ ) is normally determined by solving Equation 2.4.5 for crack length, assuming that the residual strength equals the maximum stress in the stress history. Note that the rate of growth of a crack is directly related to the rate of loss of residual strength through Equation 2.4.5, thus justifying the selection of the crack to quantify structural fatigue damage.

The critical crack length ( $a_{cr}$ ) is thus a function of material, structural geometry, and loading. As shown in [Figure 2.4.12](#), the relative effect of  $a_{cr}$  on life is typically small (i.e., when  $a_{cr}/a_0 \geq 5$ ). The primary advantage of designing for a large critical crack length is the increased inspectability it provides. A large critical crack length increases the probability of locating the crack before it becomes critical, thereby enhancing aircraft safety.



**Figure 2.4.12.** Effect of Critical Crack Size on Life

Determination of the critical crack size via Equation 2.4.5 would ordinarily be sufficient for safety limits; however, durability considerations often dictate that the final crack size,  $a_f$ , be chosen smaller than  $a_{cr}$  to represent rework or repair limits. A choice of  $a_f$  along these lines is shown in [Figure 2.4.13](#).



**Figure 2.4.13.** Economic Final Crack Size

Section 4 provides a summary of available residual strength estimating techniques and procedures that are generally applicable to all different types of structures and materials. Section

7 presents the experimental methods and procedures used to generate toughness data and residual strength data.