

11.4 Approximate Solutions for Stress Intensity Factors

This subsection discusses the procedures that one can use to obtain approximate stress intensity solutions for complicated crack problems. Approximate solutions should only be used when the objective of the damage tolerant analysis is to bound the answer and when due care has been taken to understand all aspects of the cracking behavior. Most typically, the approximate solutions are derived using known (handbook) solutions that individually account for the effects of crack geometry, global geometry and loading. As noted in subsection 11.2.1, stress-intensity factors can be added for different types of loadings when the global and crack geometries are the same. This section will concentrate on those cases where the analyst must take existing solutions for several different geometries and estimate the stress-intensity factor for the geometry of interest. In those cases where the individual geometric effects can be accounted for by multiplication of factors, the analysis is referred to as compound analysis.

There are three geometric factors that normally must be accounted for in an approximate damage tolerant analysis: stress concentration, finite width and crack shape. The effects of all three factors on the stress-intensity factor can be established exactly using careful numerical analysis procedures. However, the solution of damage tolerant problems requires more than the accurate development of the stress-intensity factor. Frequently, the growth process causes the crack to constantly change its shape which significantly complicates the crack growth life analysis.

In order to describe how the three geometrical effects can be estimated, a series of examples are presented. In each case, the approximate solutions are based on known solutions. If the actual solution is available, it is compared to the approximate solutions.

11.4.1 Effect of Stress Concentration

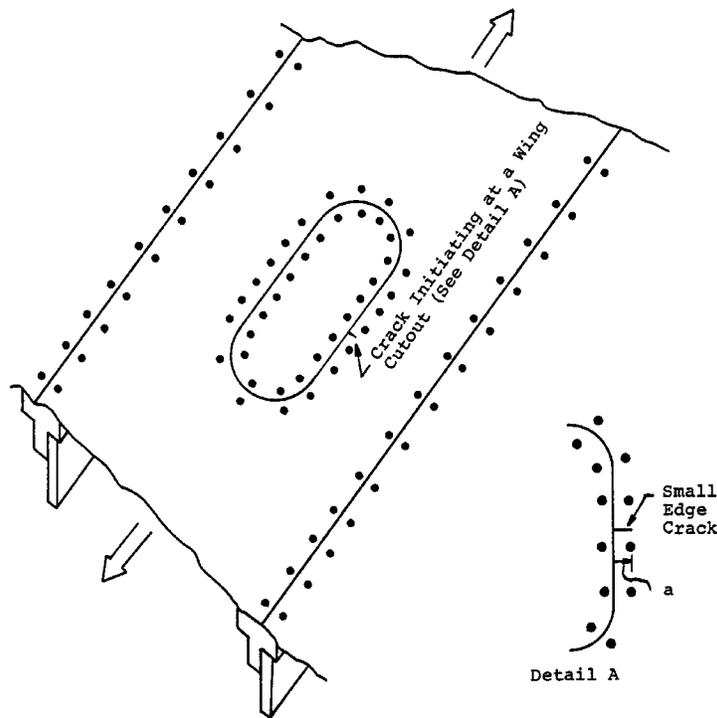
The effect of stress concentration is fairly easy to estimate for small cracks because the stress-intensity factor for an elementary crack problem can be multiplied by the elastic stress concentration factor (k_t). [Example 11.4.1](#) illustrates this point. For longer cracks initiating at stress concentrations, the crack will be propagating through the stress field created by the stress concentration and the influence of stress gradient should be taken into account. [Example 11.4.2](#) discusses an approximate method for estimating the stress intensity factor for a crack moving through a stress field generated by a stress concentration.

EXAMPLE 11.4.1 A Small Edge Crack at a Stress Concentration Site

A geometrical description of the physical problem is provided in the figure, where a small edge crack is shown growing from the edge of a wing cutout. The stress-intensity factor for an edge crack (small with respect to the element width) is found in Table 11.3.8, and is given by

$$K = \beta\sigma\sqrt{\pi a}$$

$$\beta = \sec\left(\frac{\pi a}{2W}\right) \left(\frac{\tan\left(\frac{\pi a}{2W}\right)}{\left(\frac{\pi a}{2W}\right)}\right)^{\frac{1}{2}} \left[0.752 + 2.02\left(\frac{a}{W}\right) + 0.37\left(1 - \sin\left(\frac{\pi a}{2W}\right)\right)^3 \right]$$



A Small Edge Crack Located at Stress Concentration

The stress term (σ) in the general equation typically represents the remote stress in the uniformly loaded edge cracked plate. This stress is also the stress that would exist along the line of crack propagation if no crack were present. As indicated by the figure, the stress along the line of crack propagation (assuming no crack for a moment) for the given structural configuration is the product of the remote stress and the stress concentration factor (k_t) associated with the cutout, i.e., the local stress is:

$$\sigma_{local} = \sigma \times k_t$$

For the given structural configuration, the stresses along the line of crack propagation more closely represent the type of loading that the small edge crack would experience if it were in a

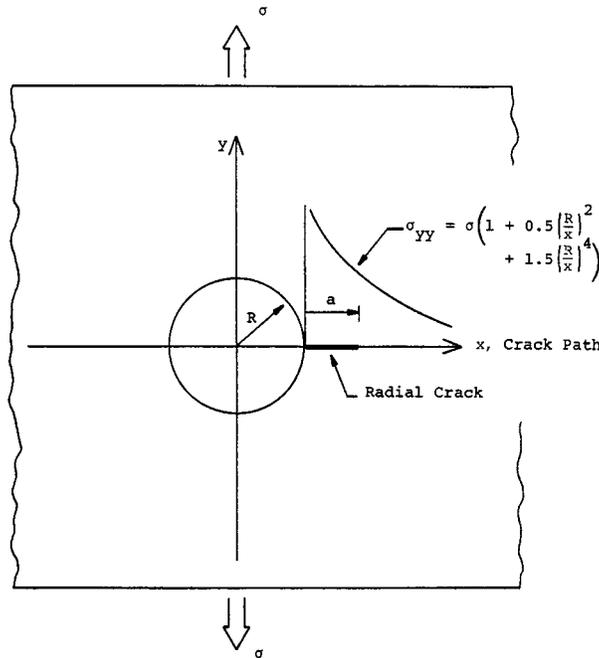
uniformly loaded edge cracked plate subjected to the higher stresses given by the equation above. It is therefore suggested that the stress-intensity factor for the structural configuration given in the [figure](#) would be close to

$$K = (\sigma \times k_t) \beta \sqrt{\pi a}$$

In general, as long as one is dealing with small edge cracks in which the width or other geometrical effects are not important, the final equation provides a reasonable approximation to the stress-intensity factor for an edge crack in the vicinity of a stress concentration. See [Example 11.4.2](#) for a discussion of stress gradient effects.

EXAMPLE 11.4.2 An Edge Crack Growing from a Stress Concentration Site

One difficulty in utilizing the [Example 11.4.1](#) final equation for cracks that emanate from a stress concentration site is that the stress concentration normally generates its own stress field. The stress concentration stress field typically exhibits the highest stresses in the vicinity of the concentration and these high stresses decay as a function of distance from the stress concentration site. The question that needs to be answered is: If the stresses along the crack propagation path are not constant, as in the case of a uniformly loaded edge cracked plate, what stresses should be used to estimate the stress-intensity factor:



Distribution of Stresses Normal to the Crack Path for a Radial Crack Growing from an Uniaxially Loaded Hole in a Wide Plate

The stress distribution associated with an uncracked hole in a wide plate is shown in the figure. As can be seen, the (normal) stress drops off rapidly as a function of distance from the edge of the hole. An evaluation of the normal stress right at the edge of the hole, i.e., the local stress, leads one to the fact that

$$\sigma_{local} = \sigma \times 3$$

(which is obtained by letting $R/X = 1$ in the equation given in the figure). Thus k_t for the uniaxially loaded hole problem is three, i.e., $k_t = 3$ and the stress-intensity factor for a very small crack of length a at the edge of the hole is

$$K = (\sigma \times 3) \beta \sqrt{\pi a}$$

One estimate of the stress-intensity factor for a longer crack would be given by the equation above; but, this estimate would be high since the stresses along the crack propagation path are noted to be dropping.

11.4.2 Effect of Finite Width

As a crack tip approaches a free edge, its stress-intensity factor rapidly increases and tends to become infinite. One can look at the width contribution as a separate (multiplicative) β factor in the same way that the width contribution affects the solution of the center-crack remotely loaded geometry.

Recall that the stress-intensity factor for a loaded panel of finite width W is given by (see Table 11.3.8)

$$K = \sigma\sqrt{\pi a} \left(\sec \frac{\pi a}{W} \right)^{1/2} \quad (11.4.1)$$

which leads one to conclude that the (multiplicative) width effect β factor required to convert the infinite plate solution to the finite width solution is

$$\beta_w = \left(\sec \frac{\pi a}{W} \right)^{1/2} \quad (11.4.2)$$

Other width correction formulations yield similar results. Most SIF solutions have the width correction included in the formulation if it is necessary. If one is aware of the need the formulation should be checked before using it in an analysis.