Problem No. UDRI-3

Title: **Structural Risk Assessment for a Multiple Element Damage Scenario**

Objective:

To illustrate the use of PROF for the calculation of the probability of load path failure given a representative three element load path.

General Description:

This sample problem illustrates the use of the PROF risk analysis computer program for evaluating the probability of failure of the chordwise joint at WS407 of the C-141 airframe given the structural status of the adjacent beam cap and splice fitting. The failure probability of the chordwise joint as a function of flight hours from a reference time is calculated using representative crack growth data, stress distributions, and crack size distributions from inspections of C-141 airframes. Since the failure probability of the joint is conditioned on the failed or intact status of the beam cap and splice fitting, the probability of failure of these elements must also be calculated and the results combined.

Topics Covered: Failure probability, conditional probabilities, multiple element damage

Type of Structure: Wing chordwise joint, beam cap, splice fitting

Relevant Sections of Handbook: Section 8

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Overview of Problem Description

In the multi-element damage (MED) scenario, two or more structural elements bridge the same load path and the damage states of the elements can interact. In this scenario, failure of selected combinations of elements may not lead to system failure, but the effects of the failures may well lead to changes in the fracture mechanics (loads or geometry factors) of the remaining elements. Thus, the probability of system failure changes when the non-critical elements fail. To evaluate the failure risks of the complete structure, the functional interaction of the structural elements must also be taken into account. PROF can provide a reasonable approximation to this potentially complex calculation.

A fault tree type of analysis is first performed to identify all of the interactive states that have an affect on the conditions leading to system failure. This step is performed external to PROF and may prove to require extensive stress and fracture mechanics analyses. These states will represent structural conditions that can be modeled by deterministic crack growth analysis. PROF can then be used to calculate the conditional probability of failure, given the potential combinations of failed and intact states of the elements. The unconditional failure probability of the complete structure is a weighted average of the conditional probabilities in which the weights are the probabilities of being in each of the states, i.e., the probability that selected elements will have failed.

It is apparent that there are, potentially, a very large number of possible combinations of structural elements that would need to be considered in the analysis of a complex structure. From the viewpoint of structural interaction, it is judged that three or four elements will generally suffice. For two elements, there are only two basic combinations: the structure will fail if either element fails (the elements are in series), or the structure will not fail if one of the elements fails (the elements are in parallel). Note in the latter case, that the crack growth properties of either element will change upon failure of the other. Even this simple multi-element structure would require four PROF runs to be combined. If there are three interacting elements, there are a total of five basic combinations of series and parallel arrangements, and many more potential analysis combinations that could require PROF runs.

Problem Statement

Failure occurs at WS405 in the C-141 airframe when the chordwise joint fractures. Since the stress levels and crack growth behavior in the chordwise joint are dependent on the intact or failed status of both the splice fitting and the beam cap, the risk analysis for WS405 must combine conditional fracture probabilities for the relevant combinations of the states of the structural details. The probability of failure at this wing station under routine operations was previously calculated by Lockheed Aeronautical Systems Company (LASC) for a single inspection interval at 31,000 spectrum hours using a Monte Carlo analysis [Cochran, et al., 1991]. The data were re-analyzed to demonstrate using PROF to calculate the failure risks for the same scenario.

The input required by PROF was provided by LASC from their evaluation of the failure risks at WS405. The input data that were used in the analyses are presented in discussed in detail in Berens [1993] and Cochran et al. [1991].

LASC performed extensive finite element analyses of the chordwise joint, splice fitting and beam cap at WS405 of the C-141 airframe. The intact or fractured status of the beam cap affects the stress levels in both the splice fitting and the chordwise joint. The intact or fractured status of the splice fitting also affects the stress levels in the chordwise joint. Thus, different crack size versus flight hour relations and different maximum stress per flight distributions are needed for the various combinations of intact and fractured beam caps and splice fittings.

Since structural failure at WS405 of the C-141 airframe occurs when the chordwise joint fractures, LASC established a fault tree, Figure UD-3.1, which isolated the fracture events that need to be evaluated in the calculation of the probability of failure of WS405 [Cochran, et al., 1991]. The fault tree of Figure UD-3.1 was restructured to demonstrate that the WS405 failure probability can be modeled as a weighted average of the probability of fracture of the chordwise joint, given the intact or failed status of the splice fitting and the beam cap. The weighing factors are the probabilities of the intact or fractured status of the splice fitting and the beam cap. The chordwise joint fracture can also be visualized in terms of the Venn diagram of Figure UD-3.2 in which the event is partitioned four mutually-exclusive sub-events.

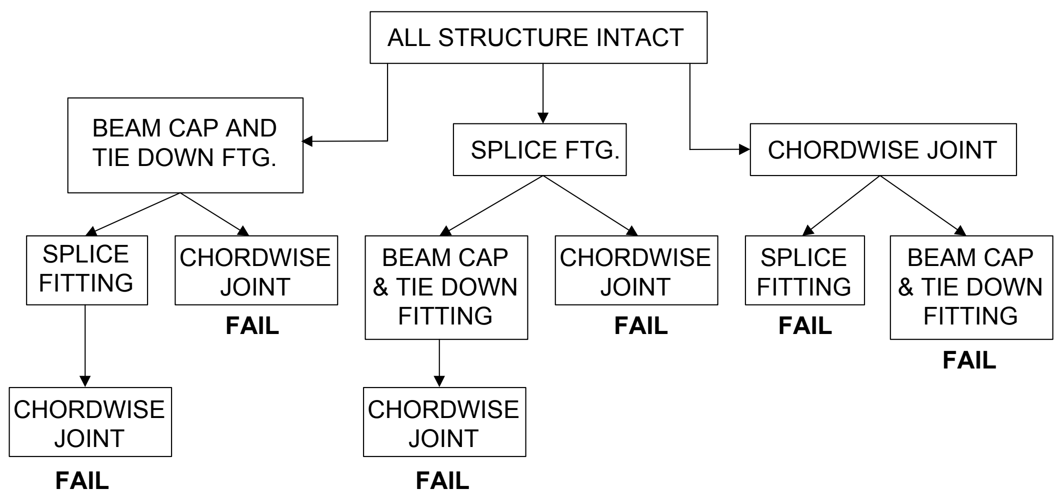

Figure UD-3.1. *WS405 Fault Tree.*

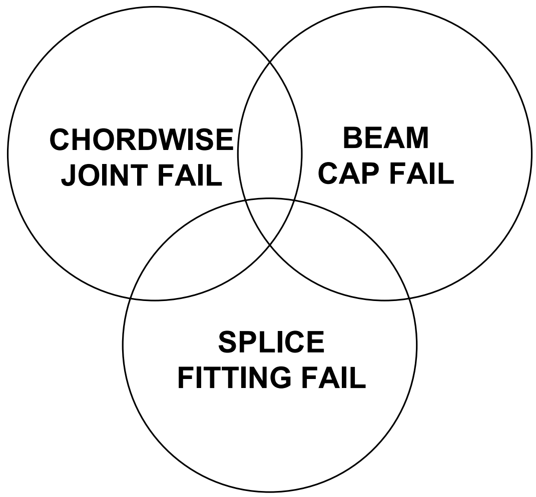

Figure UD-3.2. *WS405 Venn Diagram.*

Probabilistic Approach

The probability of failure at WS405 (POF) is given by:

$$\begin{aligned} \text{POF} &= \text{P}\{CSF,SFTAC,BCTAC\} + \text{P}\{CSF,SFTAC,BCF\} \\ &+ \text{P}\{CSF,SFF,BCTAC\} + \text{P}\{CSF,SFF,BCF\} \\ &= \text{P}\{CSF \,/\, SFTAC,BCTAC\} \bullet \text{P}\{SFTAC\} \bullet \text{P}\{BCTAC\} \\ &+ \text{P}\{CSF \,/\, SFTAC,BCF\} \bullet \text{P}\{SFTAC\} \bullet \text{P}\{BCF\} \\ &+ \text{P}\{CSF \,/\, SFF,BCTAC\} \bullet \text{P}\{SFF\} \bullet \text{P}\{BCTAC\} \\ &+ \text{P}\{CSF \,/\, SFF,BCF\} \bullet \text{P}\{SFF\} \bullet \text{P}\{BCF\}) \end{aligned} \qquad \text{(UD-3.1)}$$

where

CSF	= chordwise joint fracture
SFTAC	= splice fitting intact
SFF	= splice fitting fractured
BCTAC	= beam cap intact
BCF	= beam cap fractured
P{*A,B,C*}	= Probability of events A and B and C = $\text{P}\{A \,/\, B,C\} \bullet \text{P}\{B\} \bullet \text{P}\{C\}$
$\text{P}\{A \,/\, B,C\}$	= Conditional probability of event *A* given the events *B* and *C*

Note that because of the effect of the failed or intact effect of the beam cap on the splice fitting that

$$P\{SFF\} = P\{SF \mid BCTAC\} \cdot P\{BCTAC\} + P\{SF|BCF\} \cdot P\{BCF\} \quad \text{(UD-3.2)}$$

Further,

$$P\{SFTAC\} = 1 - P\{SFF\}$$
$$P\{BCTAC\} = 1 - P\{BCF\}. \quad \text{(UD-3.3)}$$

Time histories of the conditional probability of chordwise joint fracture given the intact or failed status of the splice fitting and beam cap were calculated using PROF (with the appropriate a versus T and maximum stress per flight distribution). Similarly, the time histories of the probability of the splice fitting and beam cap being in an intact or failed status were also calculated using PROF. These numbers were combined to calculate the unconditional probability of WS405 failure.

Selected WS405 Risk Analysis Results

PROF computed the single flight probability of fracture at ten approximately equally spaced times throughout each usage interval. The usage intervals were specified in terms of spectrum hours from the zero reference time (31,000 spectrum hours in this example) and define the times at which the inspection and repair actions are taken. In this risk evaluation at WS405 of the C-141, the analyses were performed over two usage intervals of 328-hour duration. The reported analyses were run assuming an inspection at the start of the analysis (Reference time $T = 0$ or 31000 spectrum hours).

PROF also calculates interval probability of fracture, but only at the end of a usage interval. For the structural elements and conditions of this example, the probability of fracture was dominated by cracks reaching unstable size (about 1 in.) as opposed to an encounter of a maximum stress in a flight. That is, the probability of fracture was determined primarily from the distributions of crack sizes. As a result, the single flight and interval probabilities of fracture were equal (to three significant figures) for the chordwise joint and the beam cap. The interval probabilities of fracture for the splice fitting were about five percent greater than the single flight fracture probabilities. Therefore, in this application, the single-flight fracture probabilities were used for the probabilities of intact and fractured status of the splice fitting and beam cap, Equation UD-3.1, in calculating the unconditional probability of failure at the ten times in a usage interval. This assumption is expected to occur in problems of interest because of the relatively small failure probabilities of risks in any realistic problem.

Sample results from the WS405 analysis are as follows. Figure UD-3.3 presents the probability of fracture as a function of spectrum hours for the splice fittings and the beam caps. This analysis assumed that maintenance (inspection and repair of detected cracks and failures) was performed at $T = 0$ (31,000 spectrum hours) and a subsequent maintenance was performed at 328 hours. The figure displays the relatively high fracture probabilities for the splice fittings, even after the maintenance cycle. In the original data, approximately 75 percent of the beam caps were in a failed crack size state and these were repaired before the failure probability calculations were started. The inspection capability assumed in the analysis was not sufficient to find and repair the cracks in the splice fittings. The effect of the failed beam cap on the fracture probability of the splice fitting was relatively minor in comparison to other effects.

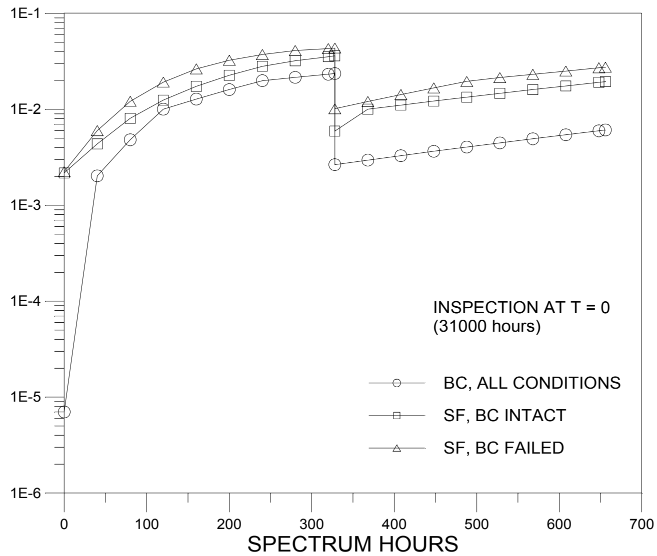

Figure UD-3.3. *Failure Probabilities of Splice Fitting and Beam Cap.*

Figure UD-3.4 presents the conditional probability of failure of the chordwise joint, given the intact or fractured status of the splice fitting and beam cap. The unconditional failure probability is a weighted average of these conditional probabilities, with the weights being determined by the proportion of intact and failed splice fittings and beam caps. Figure UD-3.5 displays the chordwise joint (system) unconditional failure probability along with the conditional failure probabilities. With the inspection at time zero, the intact or failed status of the splice fitting and beam cap had relatively minor effect on the failure probability of the system. Figure UD-3.6 compares system probabilities of failure for the analyses with and without an inspection at time zero. The effect of the maintenance action decreases the failure risks by about a factor of five.

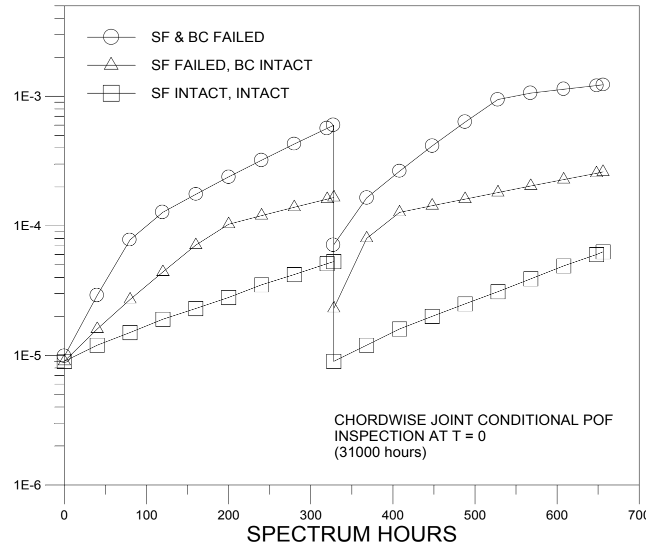

Figure UD-3.4. *Conditional Failure Probabilities of Chordwise Joint.*

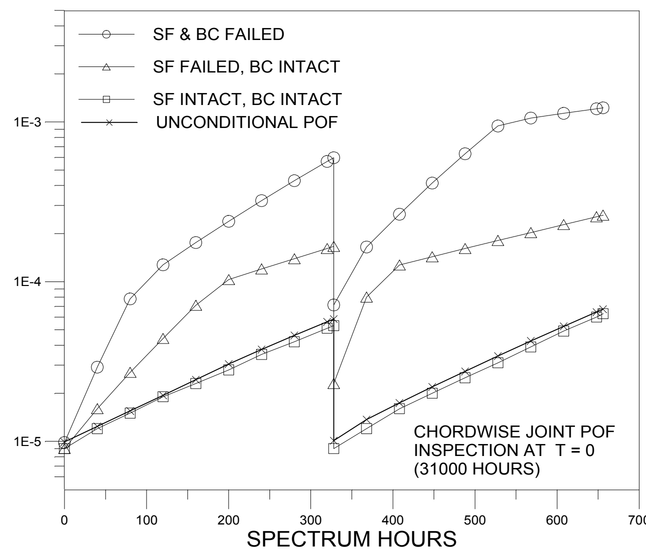

Figure UD-3.5. *Unconditional Probability of Failure of Chordwise Joint.*

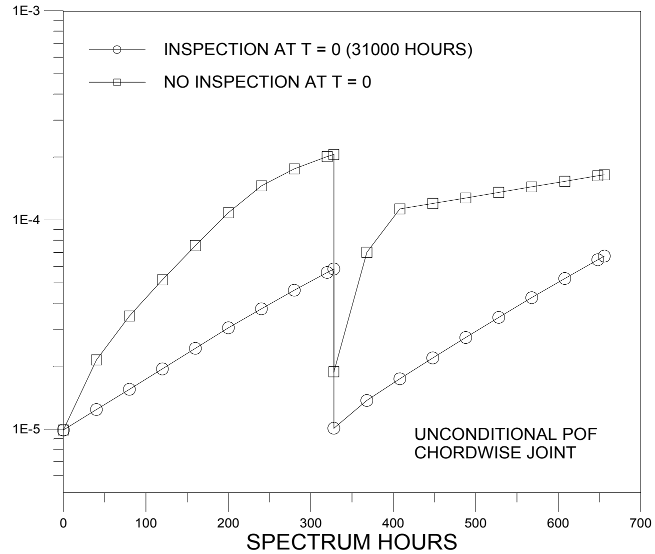

Figure UD-3.6. *Unconditional Probability of Failure of Chordwise Joint – With and Without Initial Inspection/Repair.*

Summary for Multi-Element Damage Example

The computer code PRobability Of Fracture, PROF, was used to evaluate the probability of failure at WS405 of the C-141 aircraft. Failure occurs at this location when the chordwise joint fails. The stress levels experienced by the chordwise joint are dependent on the failed or intact status of the splice fitting and the beam cap. This multi-element analysis was calculated in terms of the failure probability of the chordwise joint, given the status of the splice fitting and the beam cap, and the probabilities of the condition of the splice fitting and beam cap. The probability of failure at WS405 was calculated for a set of conditions comparable to those used in an independent analysis performed at LASC. For these conditions, the probability of a failure at WS405 in one wing was less than $2 \cdot 10^{-4}$ during a period of 656 hours of operational usage with an inspection/repair cycle at 328 hours.

References

Berens, Alan P. (1993), “Risk Analysis for C-141, WS405,” UDR-TR-93-20, University of Dayton Research Institute, Dayton, OH, 45469-0120.

Cochran, J.B., Bell, R.P., Alford, R.E., and Hammond, D.O. (1991), “C-141 WS405 Risk Assessment," *Proceedings of the 1991 USAF Structural Integrity Program Conference*, San Antonio, Texas, December, 1991.