

PROBLEM NO. MERC-4

Title: Influences of Retardation Models on Fatigue Crack Growth Predictions

Objective:

To illustrate the effects of the Willenborg and Wheeler retardation models on the prediction of fatigue crack growth behavior.

General Description:

This example problem focuses on a damage tolerance analysis of the windshield doubler at the intersection of the lower windowsill and post of an airplane. The goal is to predict the crack growth behavior of the doubler using two popular crack growth retardation models, Willenborg and Wheeler. The predicted crack growth rates are also compared to a reference case in which no retardation is applied. Stresses are obtained from a finite element model (FEM) and AFGROW is used to make the crack growth predictions.

Topics Covered: Damage tolerance analysis, crack growth analysis, retardation

Type of Structure: windows, windshield doubler

Relevant Sections of Handbook: Sections 2, 5

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Introduction

This example problem investigates the effects of two popular retardation models, Willenborg and Wheeler, on fatigue crack growth predictions. The part in question is a windshield doubler at the intersection of the upper windowsill and post of an airplane. The predicted crack growth rates are also compared to a reference case in which no retardation is applied. The retardation models will be briefly reviewed, followed by an example application to the windshield doubler. Stresses are obtained from a finite element model (FEM) of the forward fuselage and AFGROW is used to make the crack growth predictions.

Retardation Models

Retardation models address the case of reduced fatigue crack growth rates observed under variable amplitude loading conditions. They are important because fatigue crack growth measurements performed under variable amplitude loading can differ substantially from those under constant amplitude loading. During variable amplitude loading, a large loading cycle creates a large plastic zone that completely envelops the crack tip and surrounding region during subsequent smaller amplitude cycles. Retardation results from compressive residual stresses acting on the crack tip. It has been observed experimentally that crack growth is retarded as long as the plastic zone from the prior overload exceeds the plastic zone of current loading cycles. This is illustrated in [Figure MERC-4.1](#).

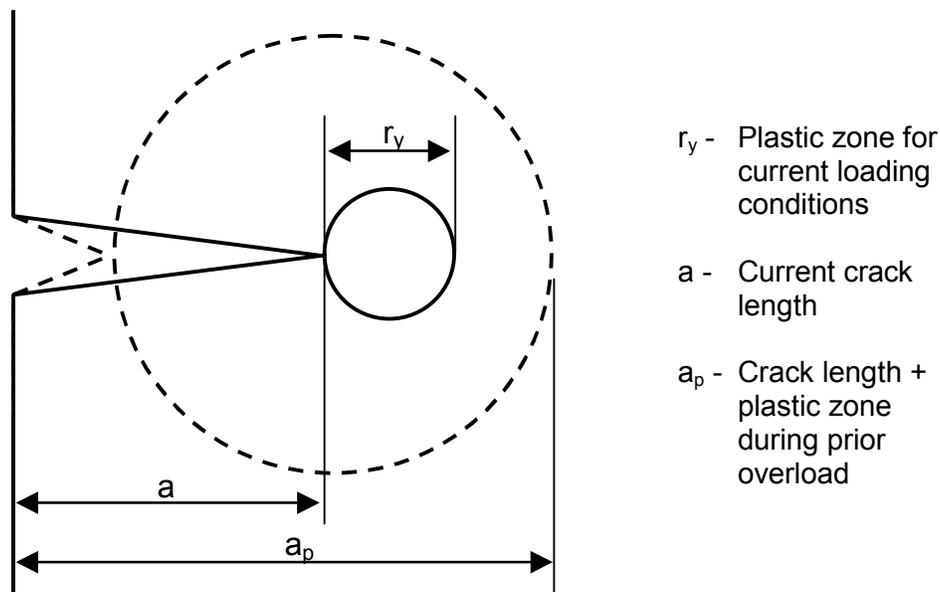


Figure MERC-4.1. Plastic zone size parameters used in crack growth retardation models.

The plastic zone size is given in Eq. (1)

$$r_y = \frac{1}{\alpha \pi} \left(\frac{K}{\sigma_y} \right)^2 \quad (1)$$

where $\alpha=2$ for plane stress and $\alpha=6$ for plane strain, K is the stress intensity factor, and σ_y is the yield stress.

Willenborg Retardation Model

The Willenborg retardation model is based on the assumption that crack growth retardation is caused by compressive residual stresses acting on the crack tip. They are represented by a single stress value, σ_{comp} , which is subtracted from both σ_{max} and σ_{min} to give corresponding effective values, $\sigma_{\text{max}}^{\text{eff}}$ and $\sigma_{\text{min}}^{\text{eff}}$.

$$\sigma_{\text{max}}^{\text{eff}} = \sigma_{\text{max}} - \sigma_{\text{comp}} \quad (2a)$$

$$\sigma_{\text{min}}^{\text{eff}} = \sigma_{\text{min}} - \sigma_{\text{comp}} \quad (2b)$$

Either effective value is set equal to zero if it is negative. The compressive stress is defined as the difference between σ_{max} and the stress required to create a plastic zone extending to the edge of a plastic zone due to a prior overload. Equation (1) is used to calculate plastic zone size.

The effective stresses are used to calculate an effective stress intensity factor range

$$\Delta K^{\text{eff}} = \beta \left(\sigma_{\text{max}}^{\text{eff}} - \sigma_{\text{min}}^{\text{eff}} \right) \sqrt{\pi a} \quad (3)$$

and an effective stress ratio

$$R^{\text{eff}} = \frac{\sigma_{\text{min}}^{\text{eff}}}{\sigma_{\text{max}}^{\text{eff}}} \quad (4)$$

that are then used in the crack growth calculations.

Wheeler Retardation Model

The Wheeler model assumes that the retardation in the crack growth rate following an overload can be obtained by scaling the constant stress amplitude growth rate according to plastic zone size. The scaling parameter, C_p is defined as

$$C_p = \left(\frac{r_y}{a_p - a} \right)^p \quad (5)$$

where p is an empirically determined constant and all other variables are defined in [Figure MERC-4.1](#).

Fatigue Crack Growth Analysis

Critical Area

The retardation models will be applied to a fatigue crack growth analysis of the windshield doubler at the intersection of the upper windowsill and window post. The doubler is shown in the finite element models in [Figures MERC-4.2](#) and [MERC-4.3](#). It is fabricated from 0.091" thick 7075-T6 aluminum and is 1.5" wide at the crack location.

Finite Element Mesh

A NASTRAN finite element model (FEM) of the forward fuselage was developed and is shown in [Figures MERC-4.2](#) and [MERC-4.3](#). It is made up primarily of shell and beam elements. In general, joints are modeled by shared-nodes; fasteners are not explicitly modeled. However, fasteners that attach the windshield doubler to airframe structure and skin are explicitly modeled with beam elements

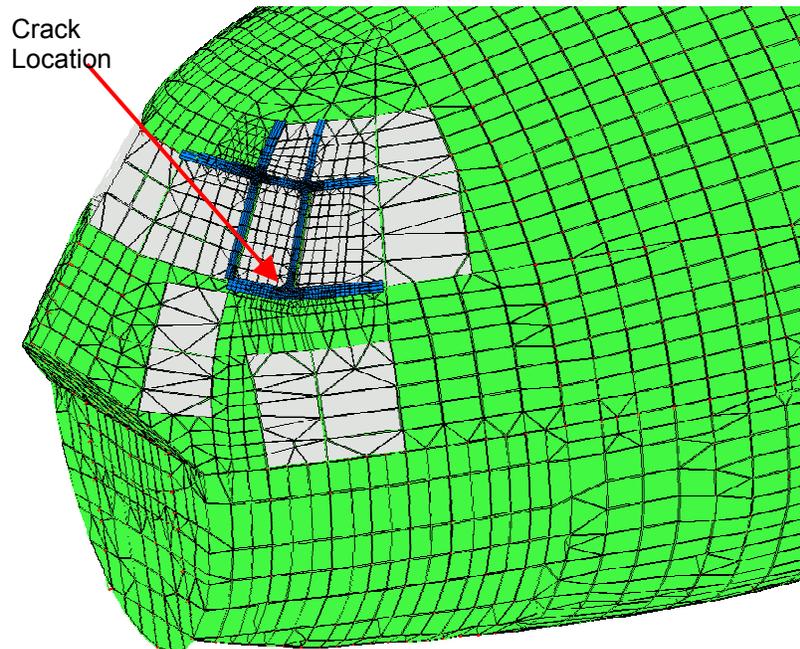


Figure MERC-4.2. Structural finite element model of forward fuselage of the aircraft with mesh refinement in front window area.

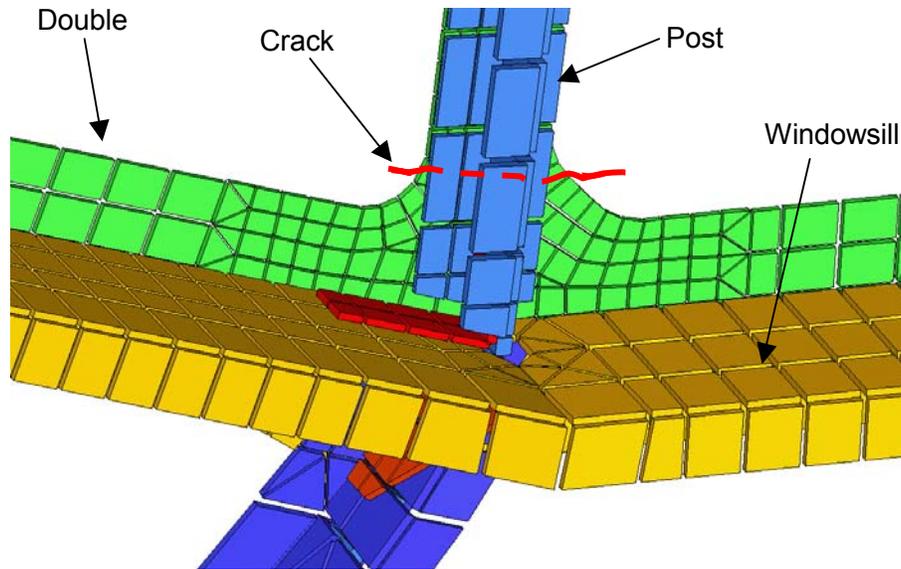


Figure MERC-4.3. Detail of structural finite element model showing mesh refinement in lower windowsill, post, and doubler. Windows are not shown. Crack occurs in doubler. View from inside of aircraft.

Stress Spectra

Internal pressurization effects are the dominant cause of stresses in the window area. Loads due to maneuvers, landings, wind gusts, etc. are negligible in comparison. Internal pressurization actually refers to the case where the cabin is maintained at sea level pressure while flying at altitudes where atmospheric pressure is substantially less.

[Figure MERC-4.4](#) shows cycles of average tensile stress in the lower leg of the doubler. High stress cycles correspond to flights at high altitudes where large pressure differentials exist between the interior of the plane and the external atmosphere. Low amplitude stress cycles correspond to low altitude flights.

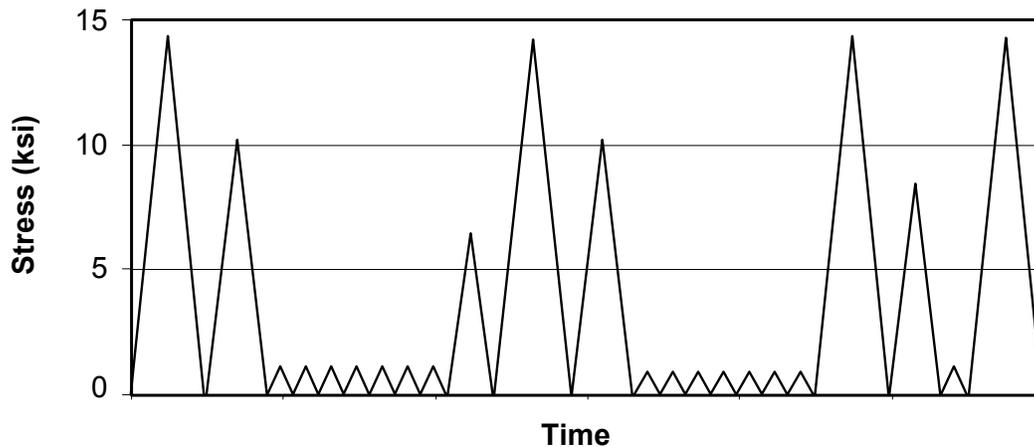


Figure MERC-4.4. Cycles of average tensile stress in lower leg of doubler.

Crack Growth Predictions and Retardation

The stress cycles in [Figure MERC-4.4](#) are clearly not constant in amplitude. Therefore, crack growth retardation mechanisms are expected to be important and significantly affect the crack growth rates.

AFGROW was used to predict the crack growth in the doubler due to fuselage pressurization cycles. The crack is assumed to start as a 0.05" radius corner crack at a fastener hole, grow to a through crack, and then grow across the doubler until failure occurs. Stress cycles, β -factors from AFGROW and FRANC2D/3D, and material da/dN data are combined to give the crack growth predictions in [Figure MERC-4.5](#) below.

Three predictions are shown on the graph. They correspond to the cases of (1) no retardation, (2) Willenborg retardation model, and (3) Wheeler retardation model with $p=1.0$. The Willenborg retardation model increases the predicted life by approximately 8% over the reference no-retardation case. The Wheeler retardation model yields an additional 8% predicted increase in fatigue life of the doubler. The retardation models do not appear to drastically alter the fundamental nature of the crack growth predictions. All

indicate that crack growth accelerates dramatically once the crack reaches approximately 0.3 inches in length.

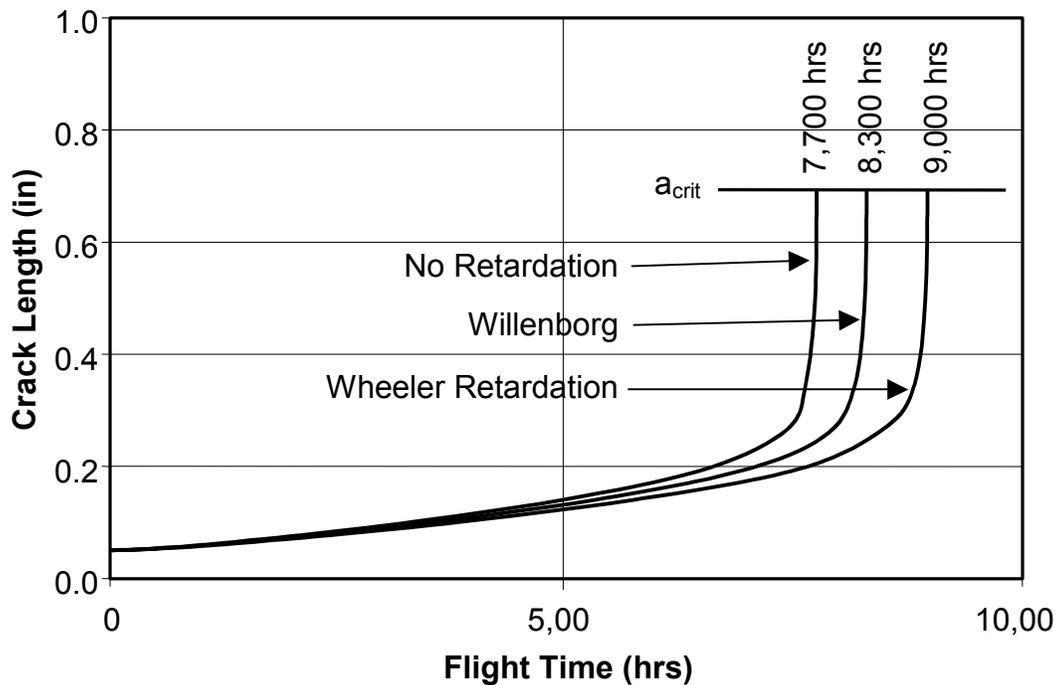


Figure MERC-4.5. Predicted crack growth versus flight hours for a crack growing across a lower portion of the windshield doubler. Effects of the Willenborg and Wheeler retardation models are compared to the reference case with no retardation.

Summary

This example problem focused on a damage tolerance analysis of the windshield doubler at the intersection of the lower windowsill and post of an airplane. The crack growth behavior of the doubler was predicted using two popular crack growth retardation models, Willenborg and Wheeler. The predicted crack growth rates were compared to a reference case in which no retardation is applied. Stresses were obtained from a finite element model and AFGROW was used to make the crack growth predictions.