PROBLEM NO. MERC-3

Title: Sensitivity of Fatigue Crack Growth Rates to Operating Conditions

Objective:

To quantify the sensitivity of fatigue crack growth rates and service life to various input parameters such as stress level and initial crack length.

General Description:

This problem focuses on quantifying the sensitivity of certain fatigue performance factors such as crack growth rate and operational service life to variations and/or uncertainties in input parameters such as cyclic stress amplitude and initial crack length. Simplified problems are first studied analytically. They are then followed by a complex application in the windshield area of a military airplane.

Topics Covered: Damage tolerance analysis, fatigue crack growth analysis, sensitivity analysis

Type of Structure: longeron

Relevant Sections of Handbook: Sections 2, 5

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Introduction

Damage tolerance analyses (DTA) of structures are often challenging and time consuming endeavors because of the types of data required and the sophistication of the techniques necessary to obtain it. Examples of required data include stress states in parts so complex that finite element (FE) analyses are necessary to obtain them. The fatigue crack growth behavior of materials may require lengthy experimental characterization.

The high demands of time and expertise can easily exceed the resources available. In such cases, it becomes necessary to prioritize the efforts devoted to obtaining various pieces of information according to each one's level of importance to the final analysis. Critical factors should receive large resources to enable their accurate determination. On the other hand, quick approximations may prove satisfactory for factors of secondary importance. But the question arises, "Which factors are critical, and which are secondary?" This example problem aims to address that question.

Mathematical Background: Sensitivity Analysis

Before one can determine what factors are and are not important in a DTA, one must choose a method of quantifying the qualitative term, *important*. Here, we have chosen to use a sensitivity analysis approach. It relates the percentage change in a system's input to the resulting percentage change in the system's output, the ratio of the two being the sensitivity parameter. As an example, consider the following equation

$$y = Ax^n \tag{1}$$

where x is the input, y is the output, and A and n are constants. The sensitivity of y with respect to x is therefore the ratio of the percentage change in y resulting from a given percentage change in x. The percentage change in the output y would be expressed as

$$100\%\left(\frac{\Delta y}{y}\right) \tag{2}$$

and likewise for the input variable x. Defining the sensitivity parameter, $S_{y/x}$, as the ratio of the percentage changes gives

$$S_{y/x} \equiv \frac{100\%\,(\Delta y / y)}{100\%\,(\Delta x / x)} \tag{3}$$

and taking the limit as $\Delta x \to 0$ gives the analytical definition of $S_{y/x}$.

$$S_{y/x} \equiv \frac{dy}{dx} \cdot \frac{x}{y} \tag{4}$$

Applying Eq. (4) to Eq. (1) gives the result

$$S_{y/x} = n \qquad (5)$$

which states that the percentage change in y is simply n times the percentage change in x regardless of the values of A and x. So if n=3 and x is increased by 10%, then y would increase 30%. This is a very useful result because of its simplicity. It will be used extensively in the following applications of damage tolerance analysis. Of course many equations exist that are not in the form of Eq. (1). In these cases, Eq. (4) must be applied on an individual basis.

Applications to Damage Tolerance Analysis

Stress Intensity Factor

One of the most fundamental steps of any DTA is calculation of the stress intensity factor, K, using Eq. (6)

$$K = \beta\sigma\sqrt{\pi a} \qquad (6)$$

where β is the geometry factor, σ is stress, and a is crack length. The sensitivity of K to the various parameters is then

$$S_{K/\beta} = 1 \qquad S_{K/\sigma} = 1 \qquad S_{K/a} = \tfrac{1}{2} \qquad (7)$$

indicating that accurate values of β and σ are equally important to the calculation of K, and that the sensitivity to crack length is less.

Crack Growth Rates

The situation becomes more interesting when crack growth rates are analyzed. A Paris Law dependence on ΔK will be assumed as follows

$$\frac{da}{dN} = C\,(\Delta K)^n = C\,(\beta\Delta\sigma)^n\,(\pi a)^{n/2} \qquad (8)$$

where N is the number of cycles, and C and n are Paris Law constants. Note that C and n are material properties with associated measurement uncertainties. Since the sensitivity of crack growth rate to the β-factor and stress is equal to n in both cases, it is worth reviewing typical values. Figure MERC-3.1 shows crack growth data for Al 7075-T6. The Paris Law forms a straight line on the logarithmic graph with n equal to the slope and C equal to da/dN at ΔK=1. It is seen that in this case, n=3.6. ($3 \le n \le$ for most materials) This value has critical implications on the accuracy of crack growth predictions. It means that a 10% error in the estimate of the β-factor results in a 36% error in the prediction of da/dN. The same sensitivity applies to the stress as well. It is this high sensitivity of da/dN to ΔK, reflected in the value of n, which presents a major challenge to the accurate prediction of crack growth rates.

What of the sensitivities to the Paris Law constants? The sensitivity to C is unity since that is its exponent in Eq. (8). It is necessary to apply Eq. (4) to Eq. (8) to determine the sensitivity to the exponent, n. Doing so gives

$$S_{\frac{da}{dN}/n} = n \ln(\Delta K) \tag{9}$$

Since ln(ΔK) is usually greater than one in engineering analyses, it is clear that the sensitivity of predicted crack growth rates to the accurate determination of the slope of the da/dN–ΔK data in Figure MERC-3.1 is even greater than to β-factors and stresses. In summary, the results are as follows

$$S_{\frac{da}{dN}/\beta} = n \qquad S_{\frac{da}{dN}/\sigma} = n \qquad S_{\frac{da}{dN}/a} = n/2$$
$$S_{\frac{da}{dN}/C} = 1 \qquad S_{\frac{da}{dN}/n} = n \ln(\Delta K) \tag{10}$$

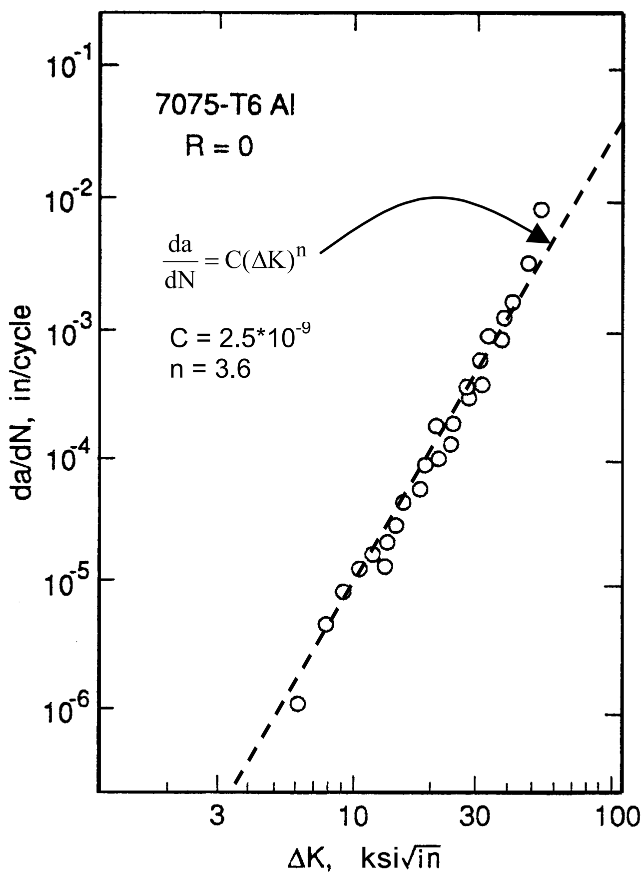

Figure MERC-3.1. da/dN – DK data for 7075-T6 Al and Paris Law curve fit.

Service Life – Cycles to Failure

The quantity of primary interest in a DTA is the service life of a component, measured in cycles to failure, N_{Life}. An analytical expression for N_{Life} can be obtained if one neglects crack retardation and assumes that the β-factor and stress range are both constant throughout a component's life. Integrating Eq. (8) and solving for N_{Life} gives

$$N_{Life} = \frac{a_f^{1-\frac{n}{2}} - a_o^{1-\frac{n}{2}}}{C\,(\beta\,\Delta\sigma\sqrt{\pi})^n \left(1 - \frac{n}{2}\right)} \tag{11}$$

where a_o is initial crack length, and a_f is final crack length at which point failure takes place. From Eq. (11), it is seen that the sensitivity of N_{Life} to certain parameters is simply negative of the crack growth rate's sensitivity to them.

$$S_{N_{Life}/C} = -1 \qquad S_{N_{Life}/\beta} = -n \qquad S_{N_{Life}/\sigma} = -n \tag{12}$$

So a 10% <u>increase</u> in the β-factor or stress would produce a 36% <u>decrease</u> in service life assuming n=3.6. Eq. (4) must be applied to Eq. (11) to determine the sensitivity of N_{Life} to initial and final crack lengths. Doing so gives

$$S_{N_{Life}/a_o} = \frac{1 - \frac{n}{2}}{1 - \left(\frac{a_f}{a_o}\right)^{1-\frac{n}{2}}} \tag{13}$$

and

$$S_{N_{Life}/a_f} = \frac{1 - \frac{n}{2}}{1 - \left(\frac{a_o}{a_f}\right)^{1-\frac{n}{2}}} \tag{14}$$

Eqs.(13) and (14) are plotted versus a_o/a_f in Figure MERC-3.2 for three values of n. The sensitivity to initial crack length depends on both n and a_f, but is approximately –1 for common values of these factors. So a 10% increase in initial crack length results in a 10% reduction in predicted fatigue life. On the other hand, predicted life is relatively insensitive to final crack length, showing only ~10% sensitivity. So a 10% increase in a_f produces only ~1% increase in predicted life. Since a_f is usually chosen to equal the critical crack length, a_{crit}, this demonstrates that variations in a_{crit} have a small impact on N_{Life} estimates.

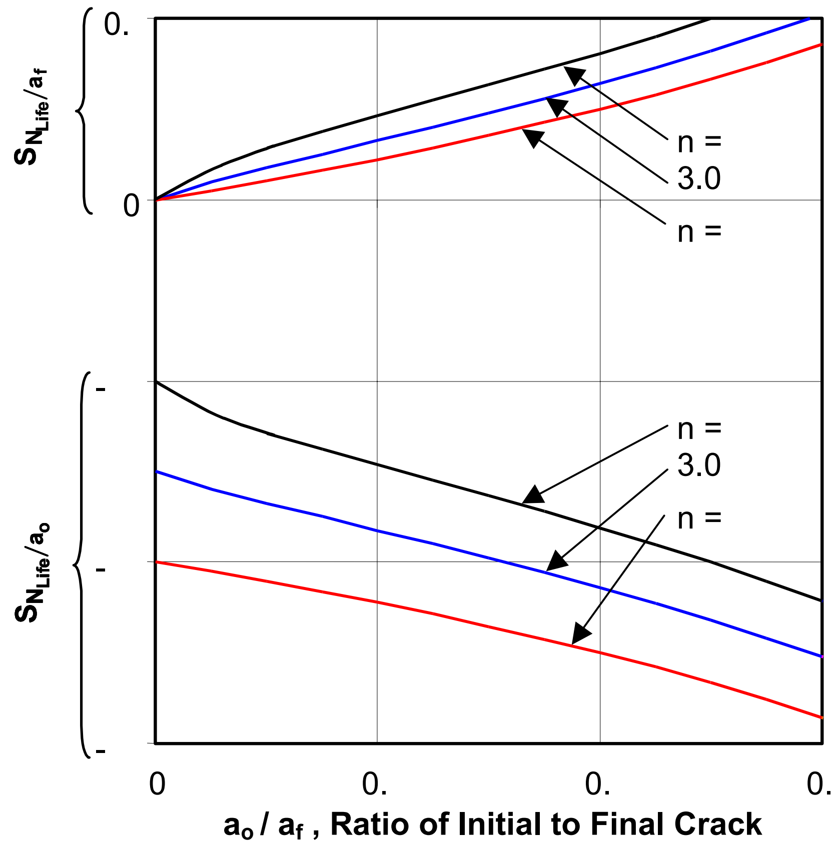

Figure MERC-3.2. Sensitivity of N_{Life} to initial and final crack lengths for three values of the Paris exponent, n. Paris law material behavior and constant β-factor and stress range are assumed.

Variable β-Factors – Numerical Example

The final example demonstrates that fatigue life sensitivity to a β-factor can depend on its relative value, with lower values being more critical than larger values. This analysis will be performed numerically rather than analytically because of the complexities of integrating non-constant β-factors. The horizontal leg of an aircraft longeron will be chosen for this example. A finite element model of it is shown in Figure MERC-3.3. The part is subjected to tension, bending, and fastener forces. The crack begins at the fastener hole and proceeds to the part edge as shown in the Figure. The β-factor is plotted in Figure MERC-3.4.

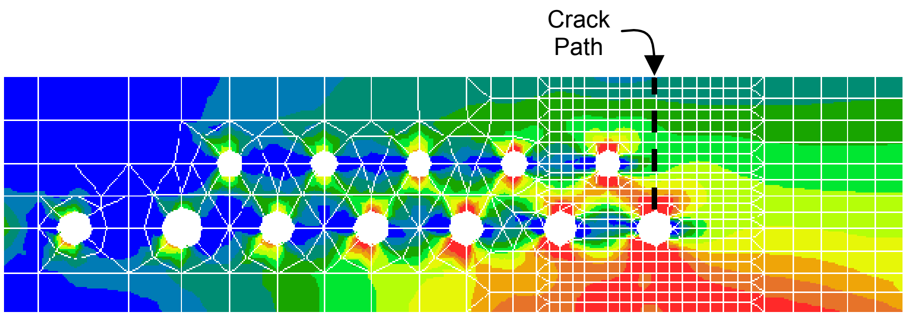

Figure MERC-3.3. Finite element model of horizontal leg of longeron. Crack originates at fastener hole and follows path shown.

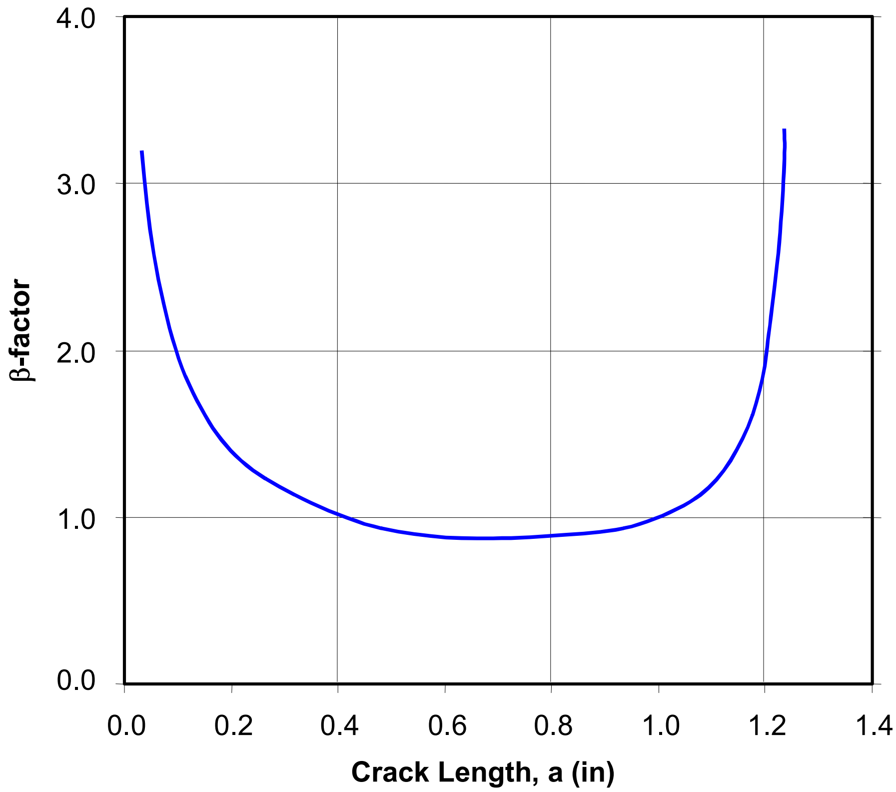

Figure MERC-3.4. β-factor versus crack length for crack starting at fastener hole in Figure MERC-3.3 and proceeding across part.

Figure MERC-3.4 shows that the β-factor is approximately three at short crack lengths because of the stress concentration at the fastener hole. The β-factor then decreases to approximately one with increasing crack length and then increases again as the crack approaches the free surface. The predicted life using the β-factor in Figure MERC-3.4 will be compared to two others having the following modifications.

Case 1. Large values of the β-factor increased. β values ≥3 were increased by 10%, β values ≤1 were not changed, intermediate values were scaled proportionately, i.e., β values = 2 were increased by 5%.

Case 2. Small values of the β-factor increased. β values ≤1 were increased by 10%, β values ≥3 were not changed, intermediate values were scaled proportionately, i.e., β values = 2 were increased by 5%.

AFGROW was used to predict the fatigue life of the part using the three different β-factor cases. Other inputs include: (1) $\Delta\sigma$=10ksi with R=0, (2) a_0=0.05 in. and a_f=1.25 in., (3) material da/dN–ΔK data taken from Figure MERC-3.1. Results are shown in Figure MERC-3.5.

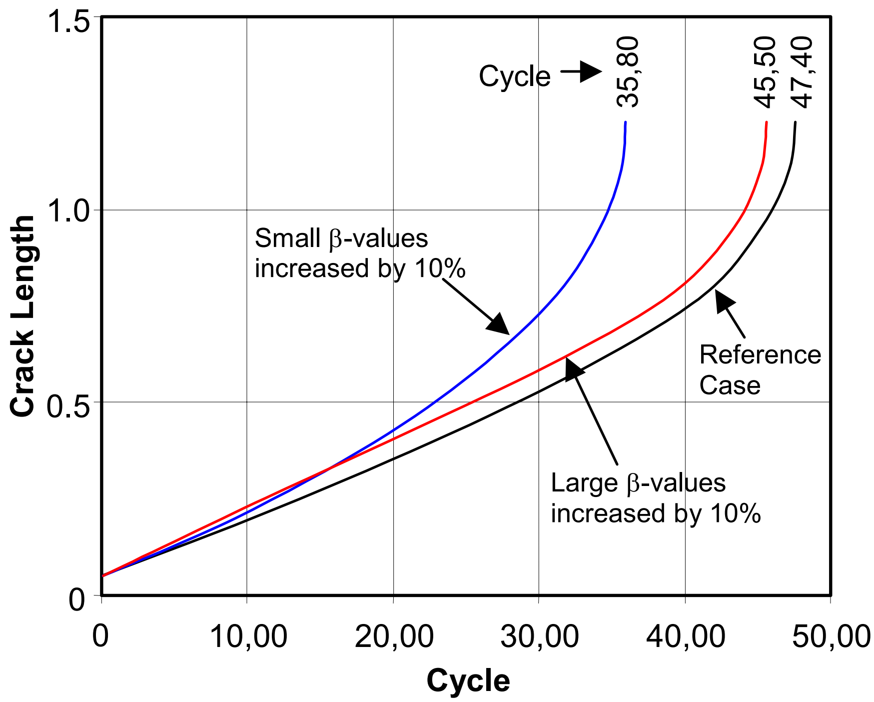

Figure MERC-3.5. Effects of β-factor variations on predicted fatigue life. Fatigue life is more sensitive to variations in small β-values than larger ones.

The 10% increase in small β-values produced a 25% decrease in predicted fatigue life, yielding a sensitivity of -2.5. The sensitivity to the increase in large β-values is -0.4. This demonstrates that fatigue life can be more sensitive to variations in small β-values than larger ones. It can therefore be more important to accurately determine small β-factor values than larger ones. This is a potentially counter-intuitive result since most analyses focus on large parameter values rather than small ones. This situation exists because cracks spend the majority of their life growing slowly at lengths with corresponding small β-factors.

Summary

A sensitivity analysis of factors affecting fatigue life predictions has been presented. It was demonstrated that certain factors have a large impact on predicted life, while others do not. Important factors include stress and β-factors. In most cases, a 10% increase in either one leads to ~35% decrease in predicted life. This high sensitivity is directly related the high sensitivity of da/dN to ΔK, which is a material property. On the other hand, factors having a relatively small impact on predicted life are critical crack length and large β-values that occur when a crack approaches a free surface.