

# PROBLEM NO. FAC-3

---

**Title:** Crack Interaction and Multi-Site Damage

**Objective**

To illustrate the process of using the finite element method to calculate stress intensity factor histories for problems involving multiple, interacting cracks resulting from multi-site damage.

**General Description:**

This problem details the process of using the finite element method to calculate stress intensity factor versus crack length histories in a flat, unstiffened panel containing a number of holes, each of which is a site for crack growth. Such histories are needed to predict fatigue crack growth rates.

*Topics Covered:* Finite element analysis, stress intensity factor calculation, crack growth, crack interaction, MSD.

*Type of Structure:* flat, unstiffened panel containing multiple holes

**Relevant Sections of Handbook:** Example FAC-1, and Sections 2, 5, 11

**Author:** Dr. A. R. Ingraffea

**Company Name:** Fracture Analysis Consultants, Inc.

121 Eastern Heights Drive  
Ithaca, NY 14850  
607-257-4970  
www.fracanalysis.com

**Contact Point:** Dr. Paul Wawrzynek

**Phone:** 607-257-4970

**e-Mail:** [wash@fracanalysis.com](mailto:wash@fracanalysis.com)

Fracture Analysis  
Consultants, Inc.  

---

## **Overview of Problem Description**

[Figure FAC-3.1](#) shows the configuration of flat, unstiffened panels tested by Luzar [1997] to examine effects of MSD on fatigue crack growth rates. 2024-T3 clad MSD panels were fabricated and tested to determine, among other objectives, fatigue crack growth values of uncorroded C/KC-135 airframe structure materials with MSD. All holes were EDM notched, and the finished EDM hole detail is shown in [Figure FAC-3.1](#) also. The panel thickness (0.063-in.), fastener hole diameter (0.25-in.) and fastener pitch (1-in.) were selected as being representative of an actual aircraft lap joint configuration. Luzar [1997] discusses the criteria used to determine the panel width, the number of open holes, and the lead crack size in more detail. The as-tested configuration of the panels with end grips resulted in a pin-to-pin load height of 70 inches and a height to width ratio of 2.9. The grip end fixtures held the test panels in double shear resulting in all of the test loads being transmitted through joint friction. This loading arrangement has been demonstrated to produce uniform stress and displacement conditions throughout the test section.

In this example a finite element model (FEM) of a representative panel is created. The FEM includes initial MSD cracks emanating from each hole. Stress intensity factors are calculated for each crack tip, and these are used to predict relative rate-of-growth of each crack. The FEM naturally includes the effects of crack interaction, and comparisons are made between the growth rates of the cracks, and between these rates and those that would occur under the simplifying assumption of no-interaction.

## **Computational Model**

A finite element model of the MSD panel, shown in [Figure FAC-3.2](#), was created using the FRANC2D/L crack growth simulator [www.cfg.cornell.edu]. Due to symmetry, only one half of the panel was modeled. The model consisted of eight-noded and six-noded plane stress elements. The model geometry and boundary conditions matched those of the experimental test panel, as shown in [Figure FAC-3.1](#). The EDM notches at the edges of each hole were modeled as simple cracks. Typical material properties for 2024-T3 clad aluminum, T-L orientation – Young’s modulus of 10.6 Msi, Poisson’s ratio of 0.33 - were used in the analysis. A uniform tensile displacement was applied at the top and bottom edges of the model to match the experiment set-up. The right hand edge of the model had free boundary conditions while the left hand edge (representing the vertical centerline) had a symmetry boundary condition. [Figure FAC-3.3](#) shows a portion of the mesh near a pair of holes. Note the refined mesh around the cracks at the edges of each hole, and the use of a uniform template of elements around each crack tip. The elements in this template are  $\frac{1}{4}$ -point singular elements. See Problem FAC-1 for guidance on meshing for accurate stress intensity factor computations.

This FEM model was used to calculate stress intensity factors at each crack tip as a function of crack length. The crack growth simulation capability of FRANC2D/L was used for this purpose. After each analysis step, FRANC2D/L calculates the stress intensity factor (SIF) values at each crack tip using the J-integral method, determines the appropriate crack growth increment for each crack, extends each crack, remeshes around each new crack tip, and then perform the next solution step with the new mesh. This procedure was performed thirteen times in order to simulate incremental crack growth.

The initial crack length used in the analysis was 0.02 inches. The fastest growing crack was incremented at steps of 0.02 inches. All other crack growth increments were automatically calculated at each step using the crack tip SIF in a power law relation with an exponent of 3.9. The final maximum crack length was, therefore, 0.26 inches. However, the final length of each crack varied with its stress intensity factor history. The analysis was arbitrarily stopped at this point since the crack lengths had far exceeded those found in the test specimens.

The initial FEM model contained about 19000 degrees-of-freedom, the final mesh about 25000. Solution time on a 1GHz Pentium III PC was about one minute per crack growth step.

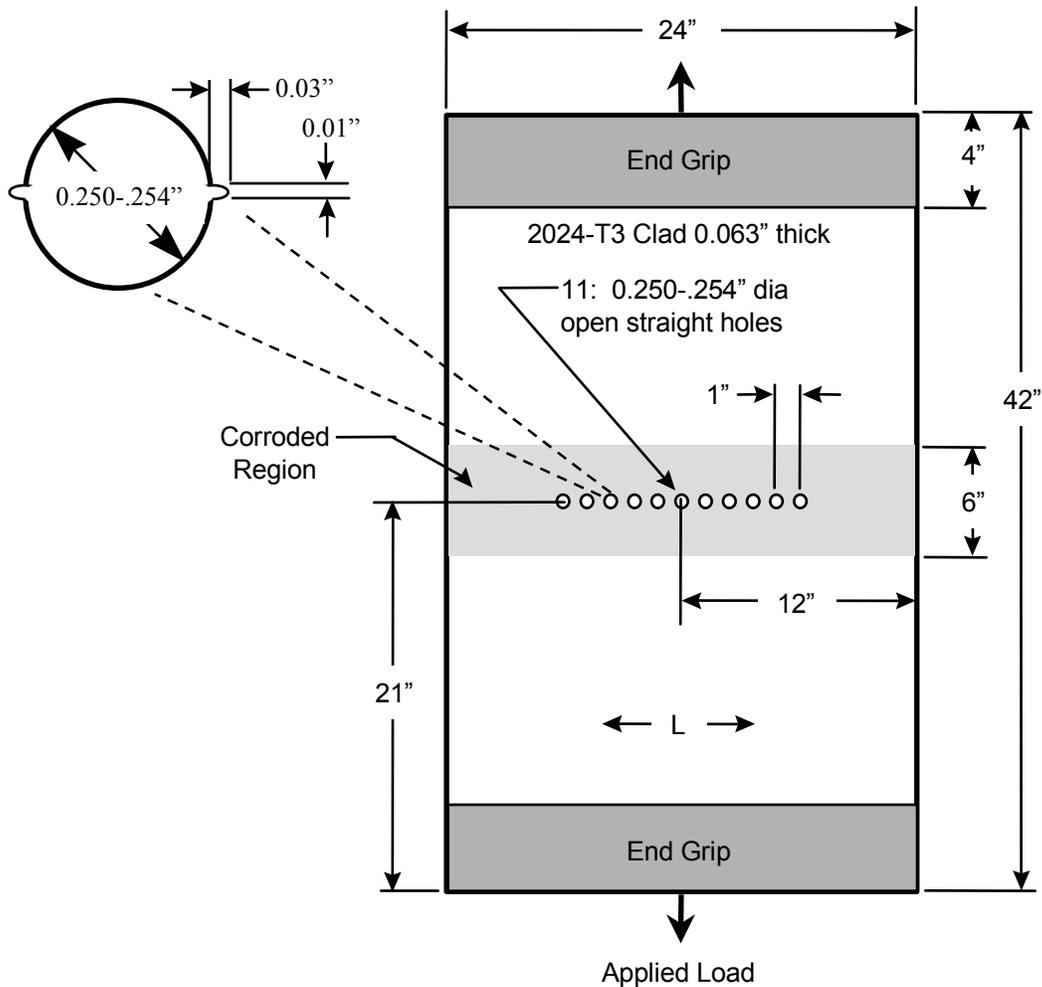
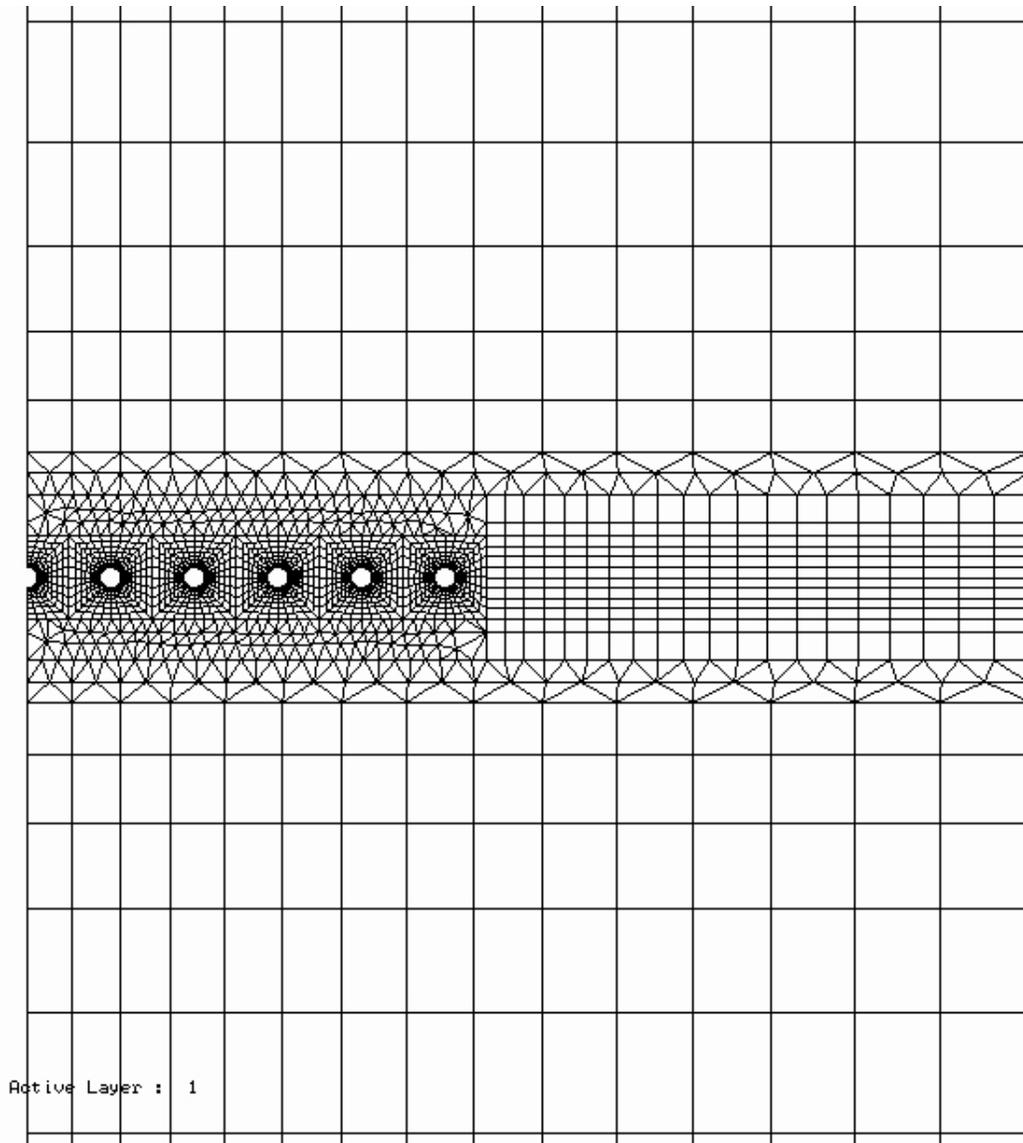
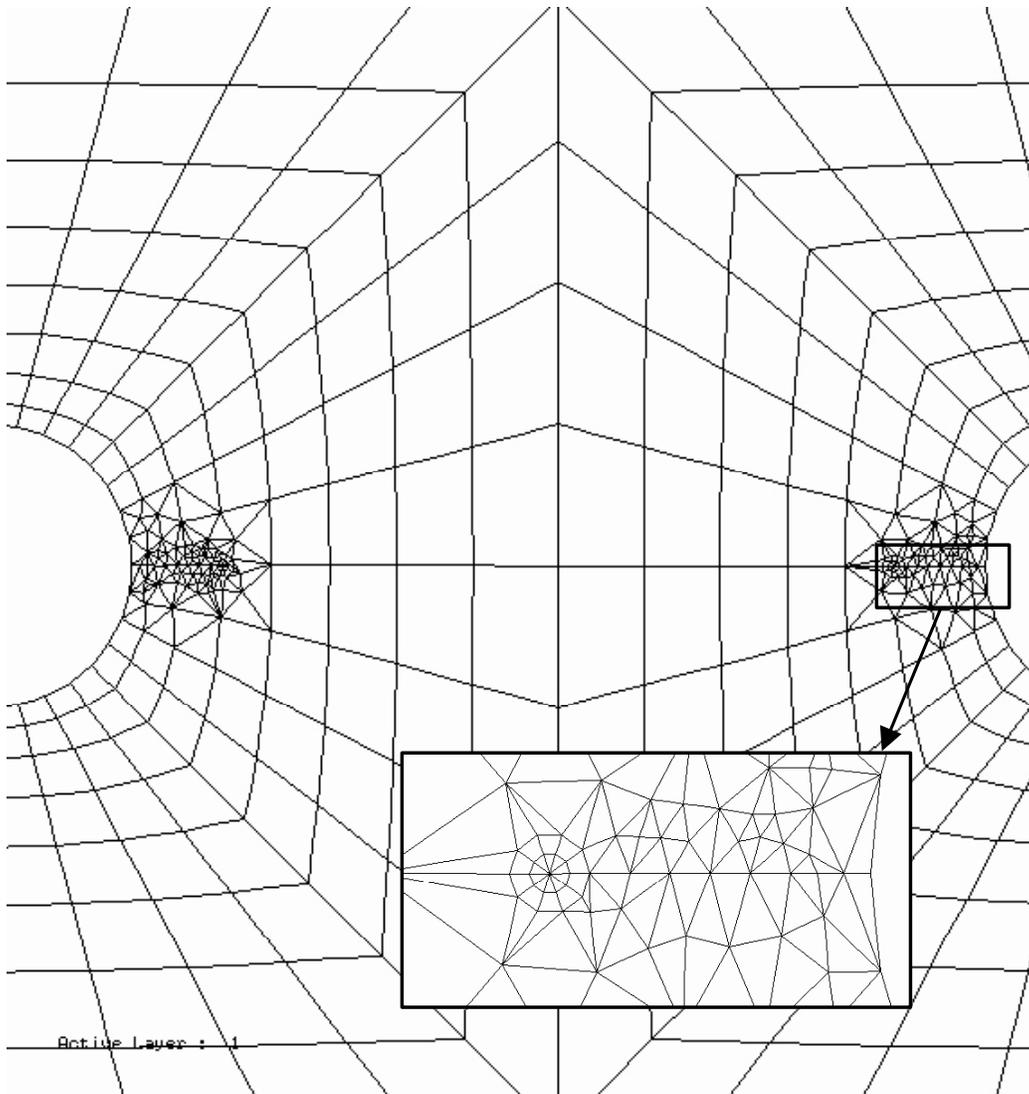


Figure FAC-3.1. MSD Panel Geometry used in this example. From Cope [1998].



*Figure FAC-3.2. Central portion of finite element model of flat plate with MSD.*



*Figure FAC-3.3. Typical details of finite element model of flat plate with MSD. Near-hole meshing after four steps of crack propagation.*

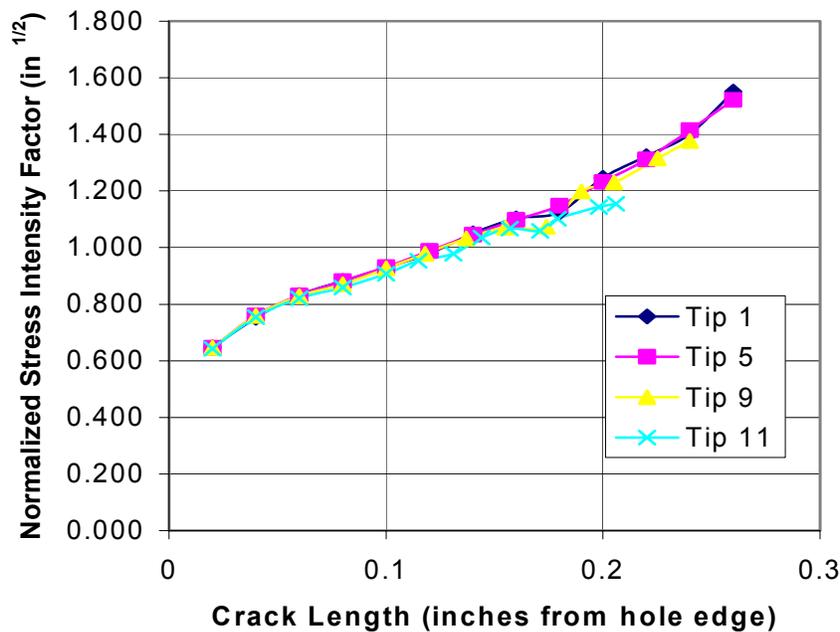
### **Computational Results**

SIF histories of crack propagation were generated for each crack tip. As shown in [Figure FAC-3.4](#), these histories were almost identical for the first 7 to 8 crack tips (numbered left to right), and for short crack lengths. There is a "falloff" of stress intensity for the rightmost 3 cracks, because of less interaction with surrounding cracks and holes. This falloff intensifies with crack growth, as would be expected with increasing crack interaction effect. At the 12<sup>th</sup> growth step, the SIF for the rightmost crack tip is about

20% less than the average of the interacting crack tips. This means that this is the slowest growing crack, as seen in [Figure FAC-3.4](#).

Another way of assessing the effects of crack interaction is shown in [Figure FAC-3.5](#). Here the stress intensity factor histories for crack tip 1, where one expects strong interaction effects, and crack tip 11, where intensification from interaction is expected to be less than that for all other cracks, are compared to the benchmark solution for non-interacting cracks. This solution [Newman 1971] is for equal-length diametrically opposed cracks emanating from a single hole in an infinite plate. [Figure FAC-3.5](#) shows very strong interaction effects present in this MSD problem with the stress intensity factor for crack 1 (and, therefore, the first 7 or 8 cracks) reaching a value more than 40% higher than that assuming no interaction.

The stress intensity factor histories shown in [Figure FAC-3.4](#) can be used to predict fatigue crack growth rates by using them as input data for AFGROW or NASGRO, as shown in example problem FAC-1. Cope *et al.*[1998] used this procedure to produce a comparison between predicted and observed crack growth behavior in the problem shown in [Figure FAC-3.1](#). This comparison is shown in [Figure FAC-3.6](#).



*Figure FAC-3.4. Predicted normalized stress intensity factor histories for representative crack tips in this MSD problem. Crack tips are numbered from left to right.*

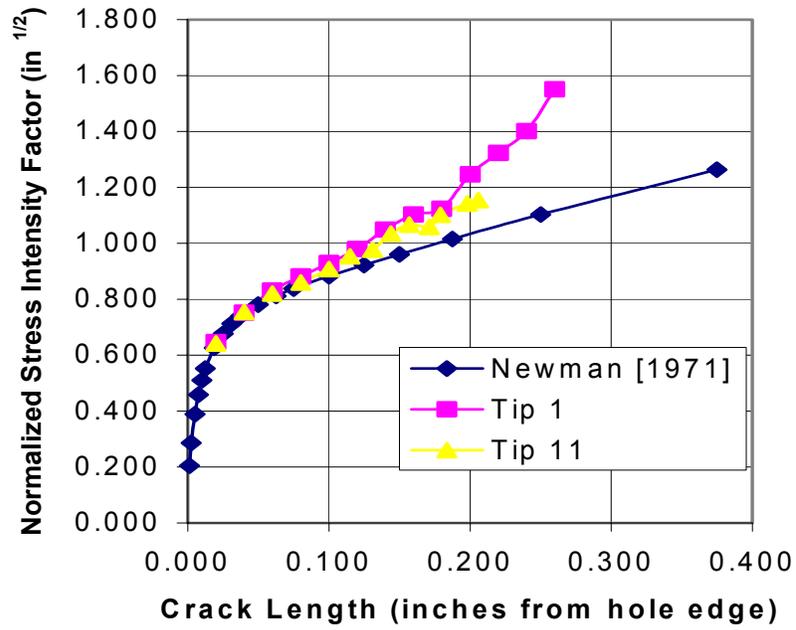


Figure FAC-3.5. Predicted normalized stress intensity factor histories for representative crack tips in this MSD problem compared to solution for non-interacting cracks. Crack tips are numbered from left to right.

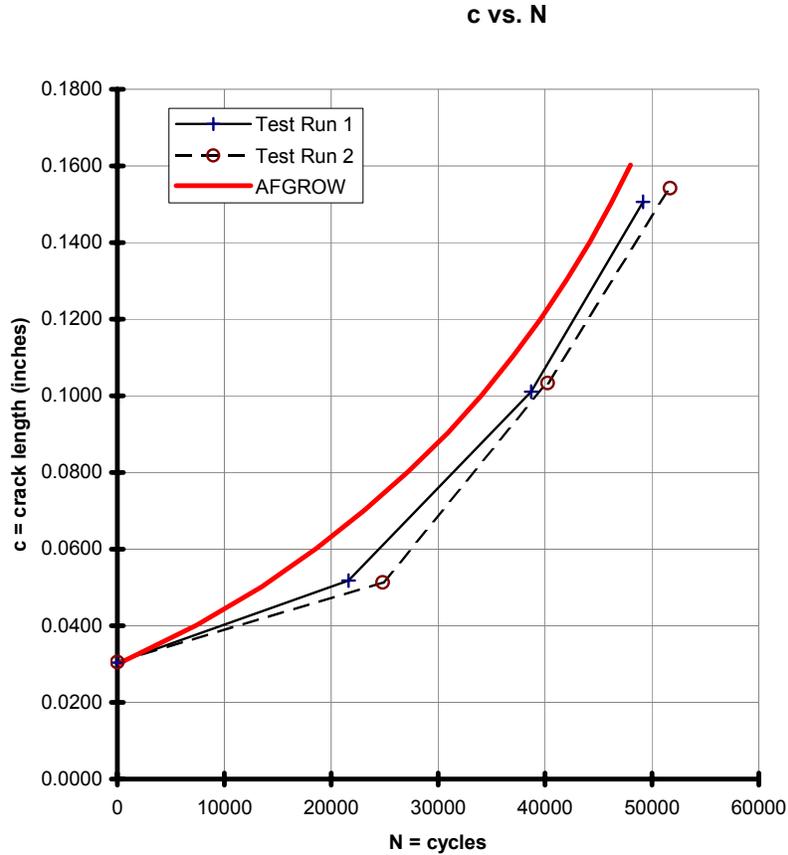


Figure FAC-3.6. Comparison between predicted and observed fatigue crack growth behavior for the problem shown in [Figure FAC-3.1](#). From Cope et al. [1998].

## **References**

Cope, D., West, D., Luzar, J., Miller, G. Corrosion Damage Assessment Framework: Corrosion/Fatigue Effects on Structural Integrity. Technical Report D500-13008-1, The Boeing Defense and Space Group, 1998.

Luzar, J.J. Pre-corroded Fastener Hole Multiple Site Damage Testing, Final Report, EA 96-135OTH-041, Boeing ISDS Post-Production/Derivative Aircraft Division, December 1997.

Newman, J. C. An Improved Method of Collocation for the Stress Analysis of Cracked Bodies with Various Shaped Boundaries. *NASA TN D-6376*, pp 1-45, 1971.